Simulating train-tunnel aerodynamics with a parallel adaptive Cartesian method

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June 5, 2019

Outline

Adaptive Cartesian finite volume methods
- Block-structured AMR with complex boundaries
- Parallelization approach

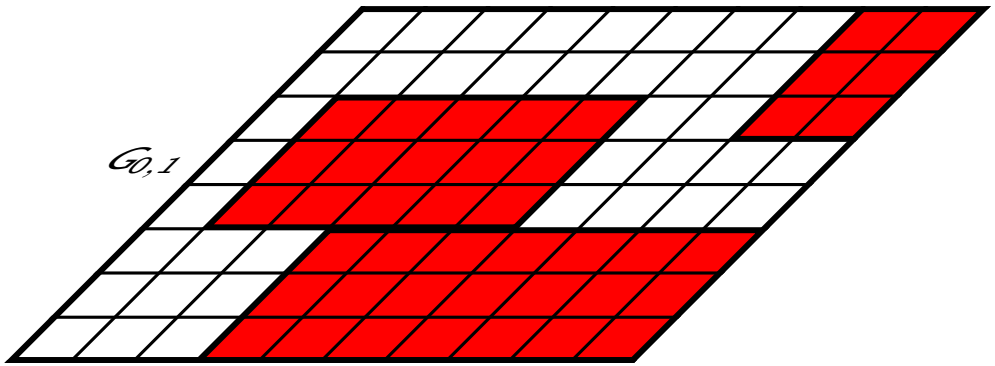
Train-tunnel aerodynamics
- Validation
- Passing trains in open space
- Passing trains in a double track tunnel

Conclusions
- Summary and outlook

Block-structured adaptive mesh refinement (SAMR)

For simplicity $\partial_t \mathbf{q}(x, y, t) + \partial_x \mathbf{f}(\mathbf{q}(x, y, t)) + \partial_y \mathbf{g}(\mathbf{q}(x, y, t)) = 0$

- Refined blocks overlay coarser ones



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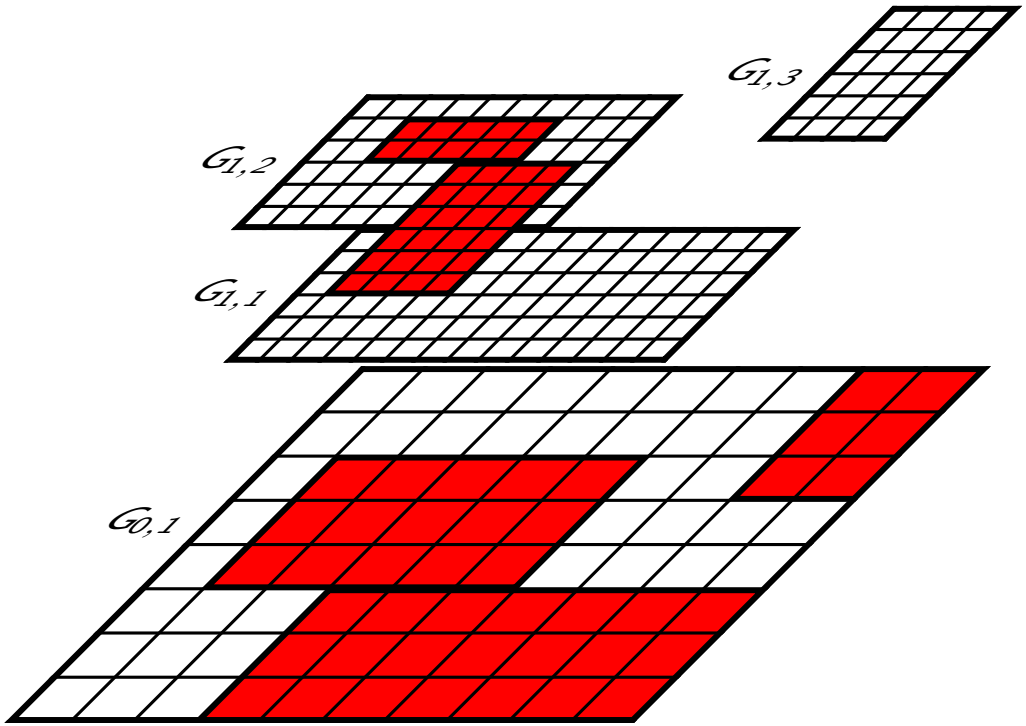

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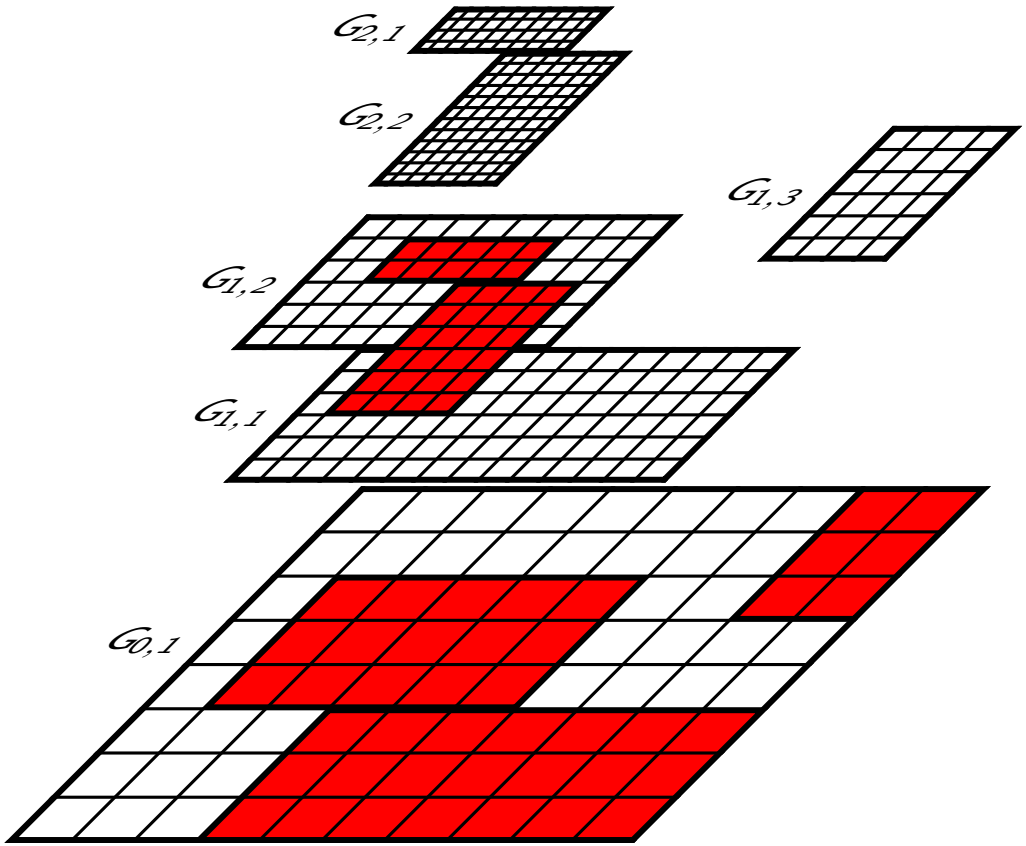

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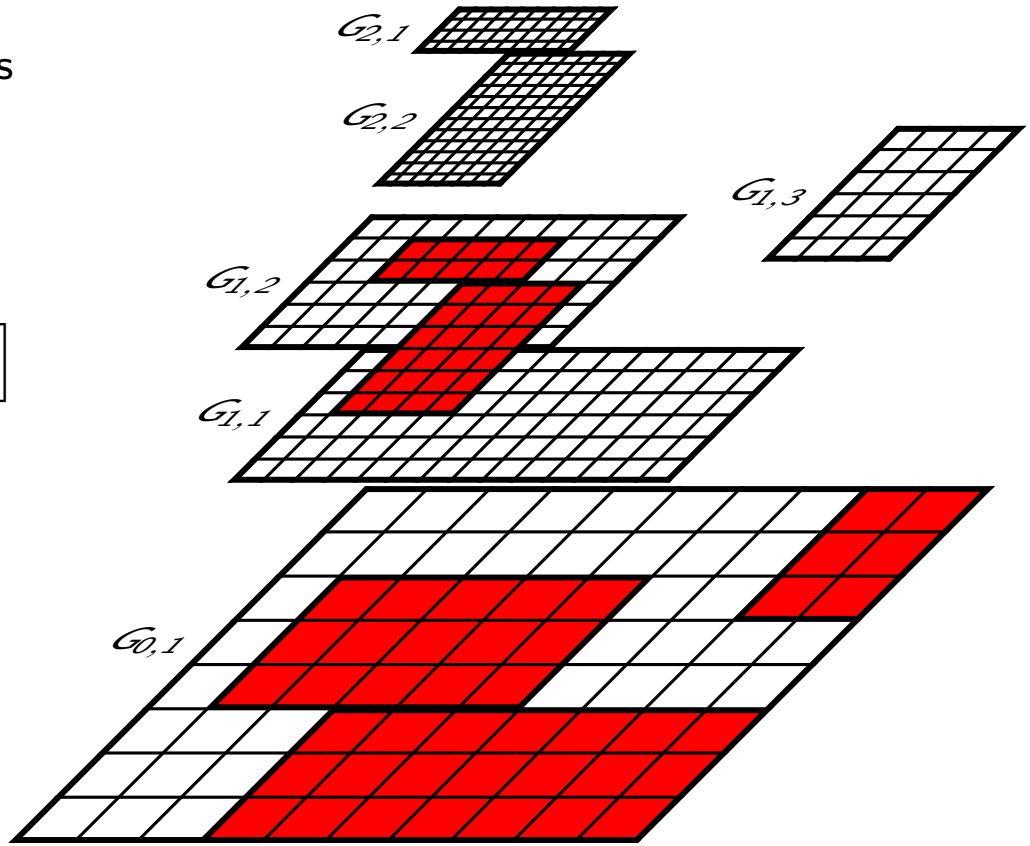
- Refined blocks overlay coarser ones
- Refinement in space *and time* by factor r_l [Berger and Colella, 1988]
- Block (aka patch) based data structures
- \+ Numerical scheme

$$\mathbf{Q}_{jk}^{n+1} = \mathbf{Q}_{jk}^{n} - \frac{\Delta t}{\Delta x}\left[\mathbf{F}_{j+\frac{1}{2},k} - \mathbf{F}_{j-\frac{1}{2},k}\right] - \frac{\Delta t}{\Delta y}\left[\mathbf{G}_{j,k+\frac{1}{2}} - \mathbf{G}_{j,k-\frac{1}{2}}\right]$$

only for single patch necessary



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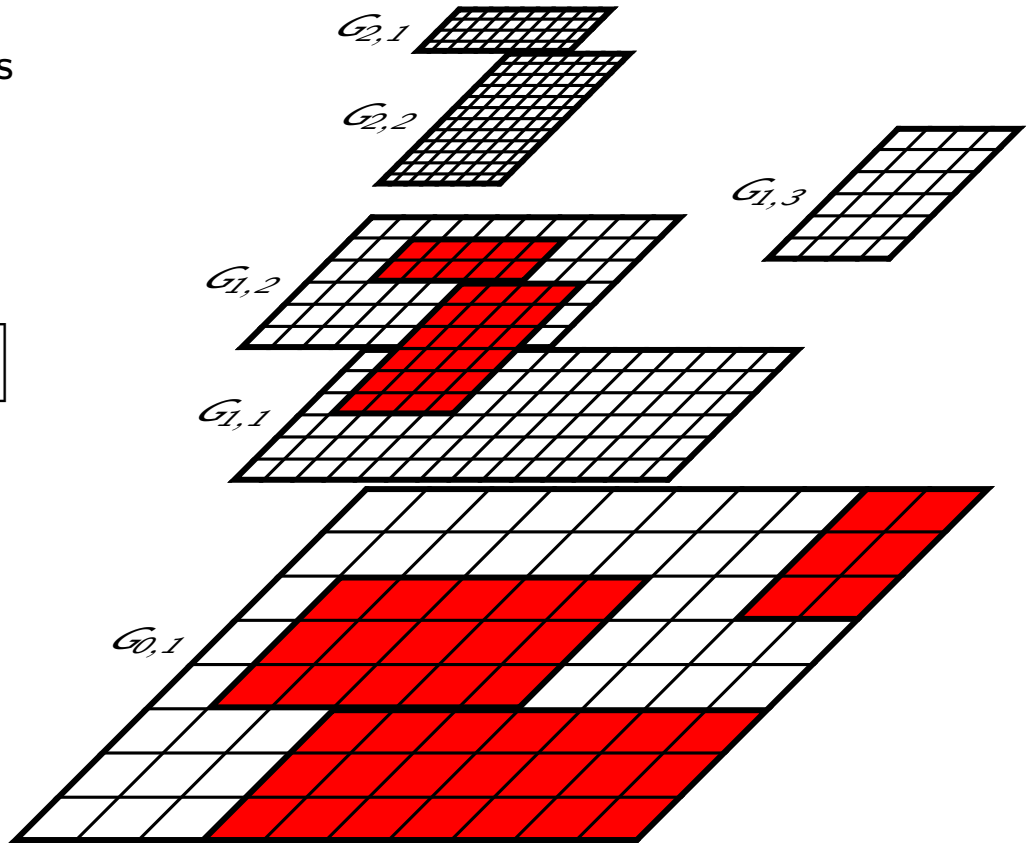
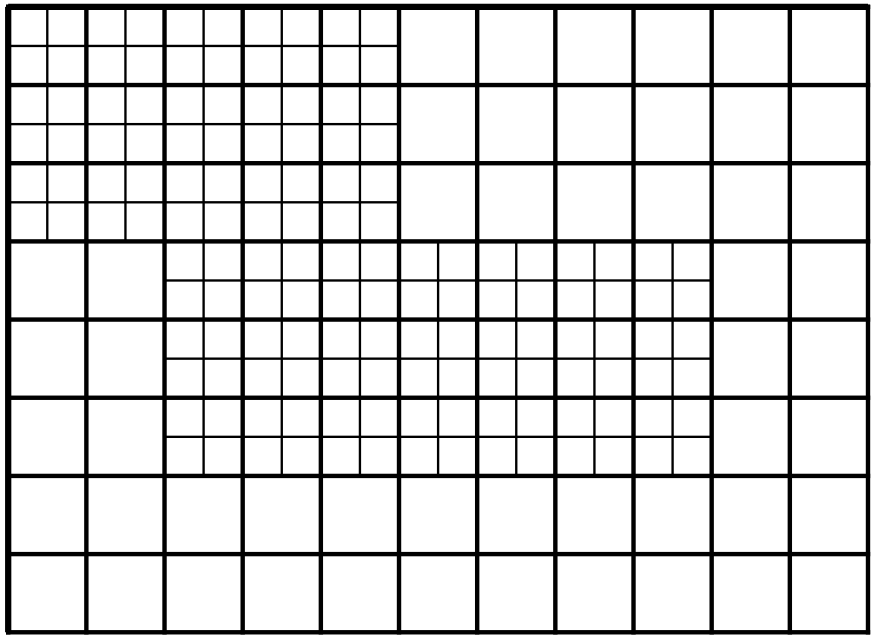
- \+ Efficient cache-reuse / vectorization possible
- \- Cluster-algorithm necessary
- Papers: [Deiterding, 2011a, Deiterding et al., 2009b, Deiterding et al., 2007]



Level transfer / setting of ghost cells

Conservative averaging (restriction):

$$\hat{\mathbf{Q}}^{l}_{jk} := \frac{1}{(r_{l+1})^2} \sum_{\kappa=0}^{r_{l+1}-1} \sum_{\iota=0}^{r_{l+1}-1} \mathbf{Q}^{l+1}_{v+\kappa,w+\iota}$$



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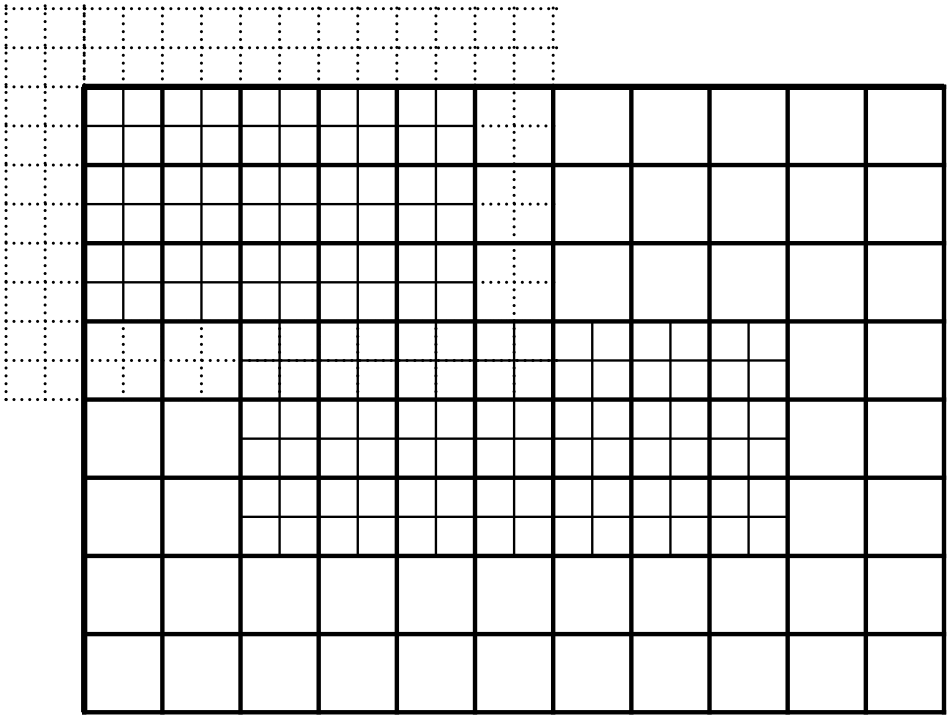

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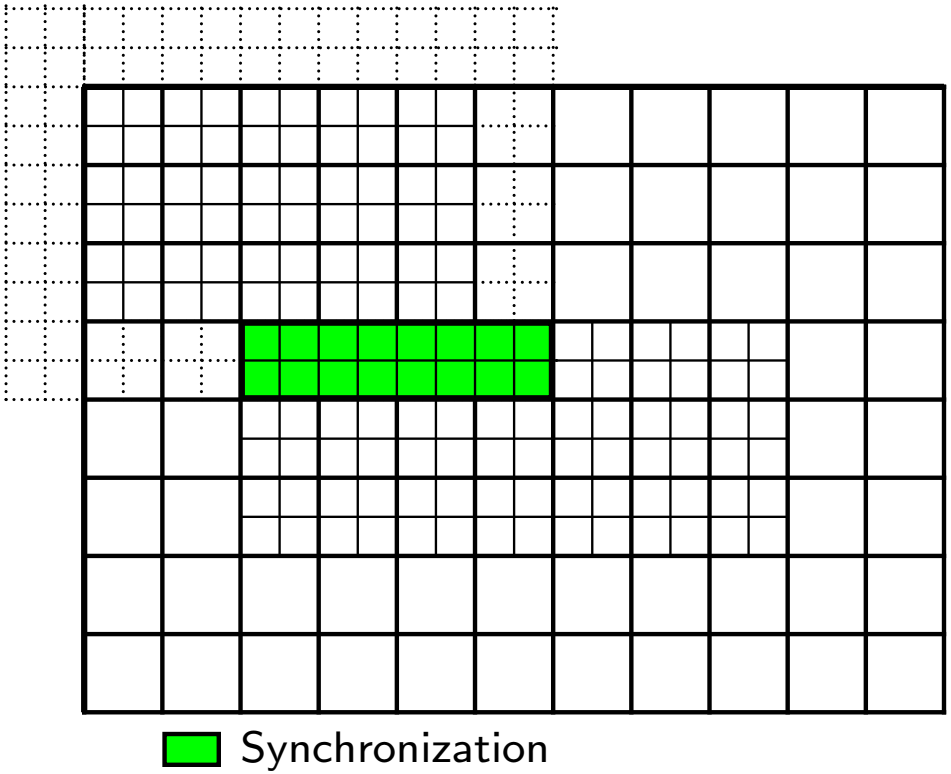

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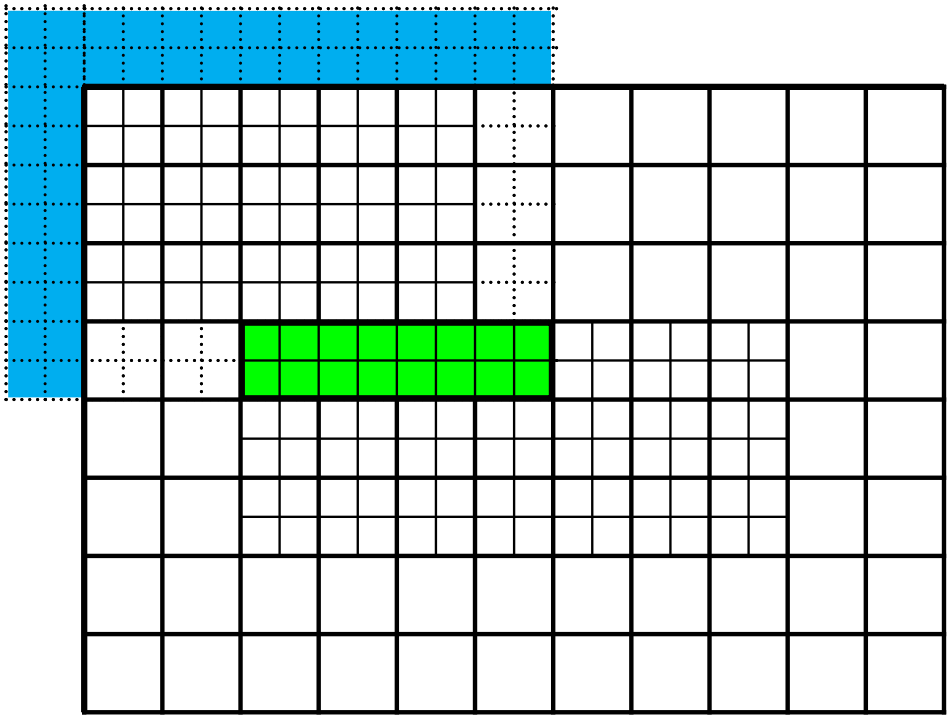

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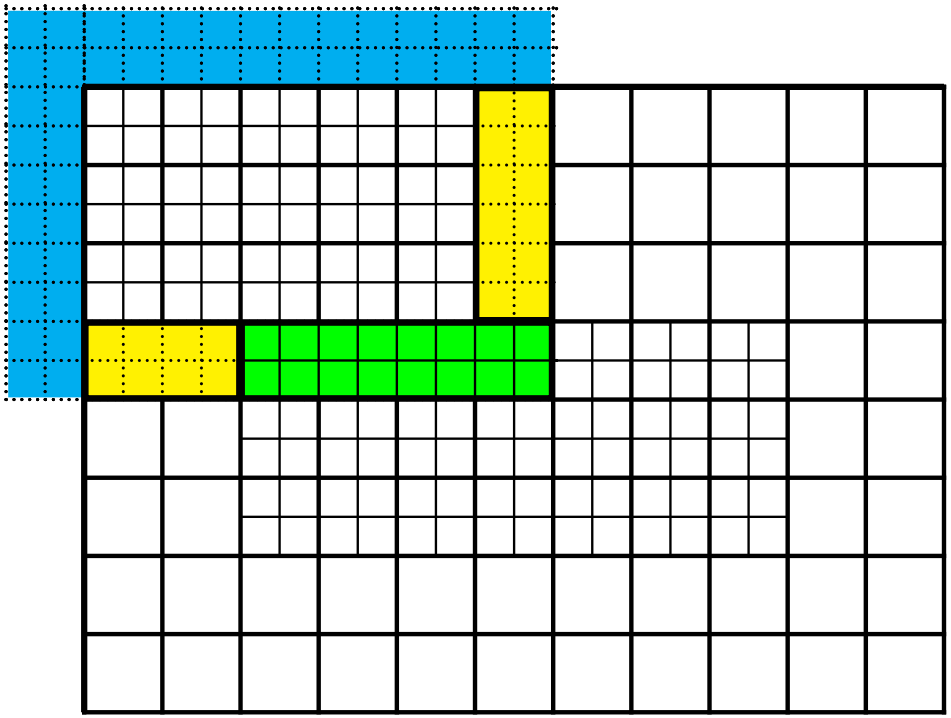
Bilinear interpolation (prolongation):

$$\begin{aligned}\check{\mathbf{Q}}^{l+1}_{vw} := {} & (1-f_1)(1-f_2)\,\mathbf{Q}^{l}_{j-1,k-1} \\ & + f_1(1-f_2)\,\mathbf{Q}^{l}_{j,k-1} + \\ & (1-f_1)f_2\,\mathbf{Q}^{l}_{j-1,k} + f_1 f_2\,\mathbf{Q}^{l}_{jk}\end{aligned}$$

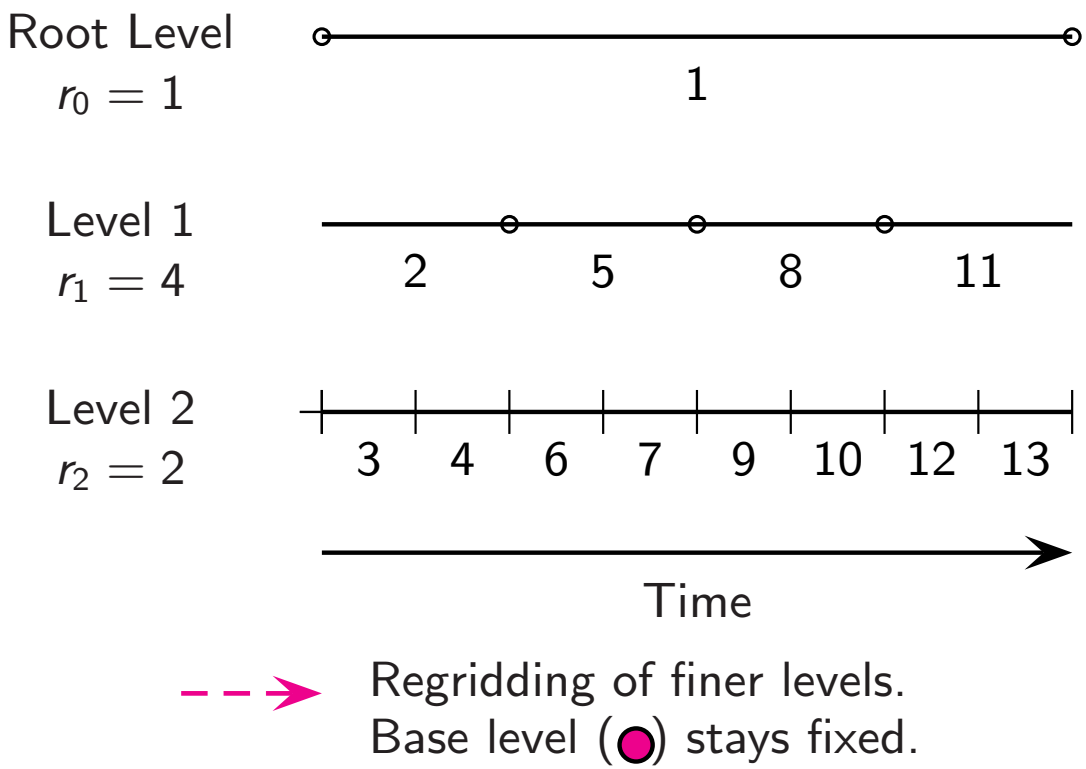


For boundary conditions: linear time interpolation

$$\tilde{\mathbf{Q}}^{l+1}(t+\kappa\Delta t_{l+1}) := \left(1-\frac{\kappa}{r_{l+1}}\right)\check{\mathbf{Q}}^{l+1}(t) + \frac{\kappa}{r_{l+1}}\check{\mathbf{Q}}^{l+1}(t+\Delta t_l) \quad \text{for } \kappa = 0,\ldots r_{l+1}$$

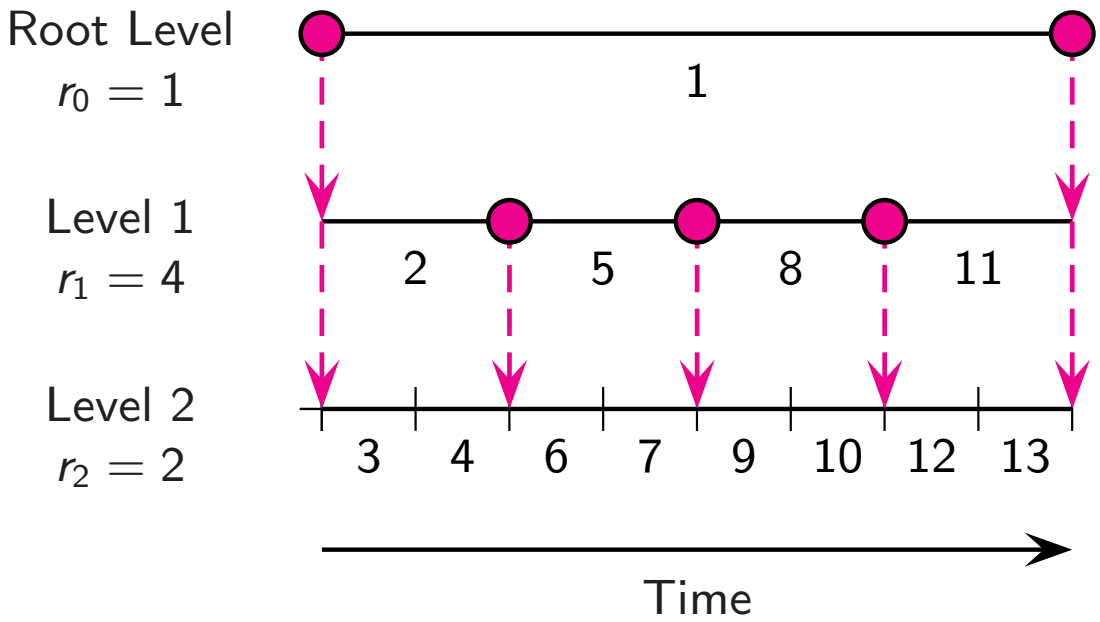
Recursive integration order

- Space-time interpolation of coarse data to set $I_l^s, l > 0$

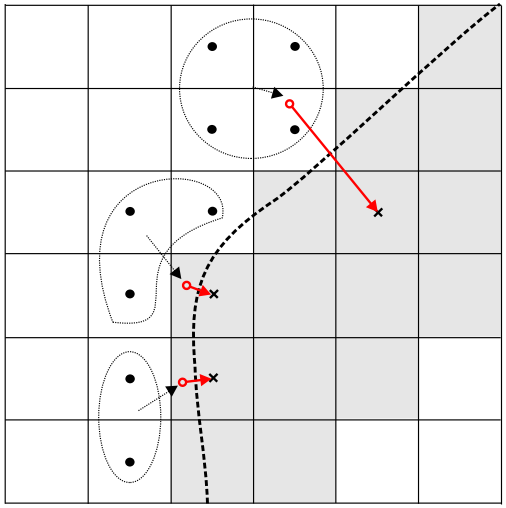


Recursive integration order

- Space-time interpolation of coarse data to set $I_l^s, l > 0$
- Regridding:
  - Creation of new grids, copy existing cells on level $l > 0$
  - Spatial interpolation to initialize new cells on level $l > 0$



Level-set method for boundary embedding



- Implicit boundary representation via distance function φ, normal $\mathbf{n} = \nabla\varphi/|\nabla\varphi|$
- Complex boundary moving with local velocity $\mathbf{w}$, treat interface as moving rigid wall [Deiterding et al., 2007]
- Construction of values in embedded boundary cells by interpolation / extrapolation [Deiterding, 2009, Deiterding, 2011a]
- Creation of level set from triangulated surface data with closest-point-transform (CPT) algorithm [Mauch, 2003, Deiterding et al., 2006]

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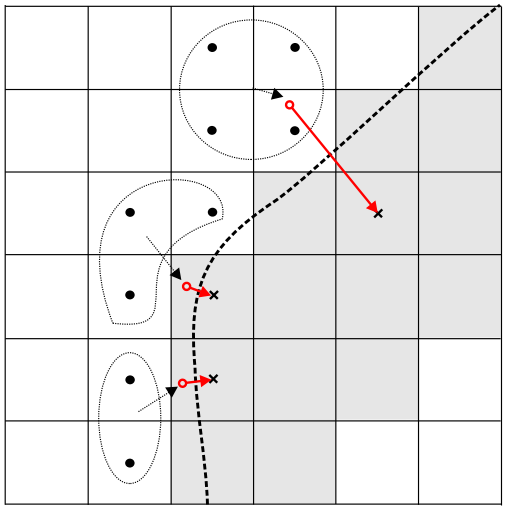

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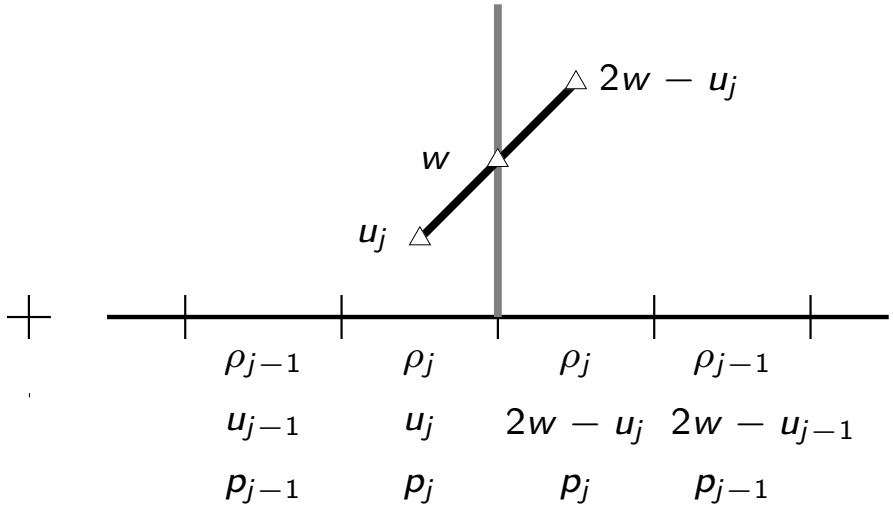
Interpolate / constant value extrapolate values at

$$\tilde{\mathbf{x}} = \mathbf{x} + 2\varphi\mathbf{n}$$



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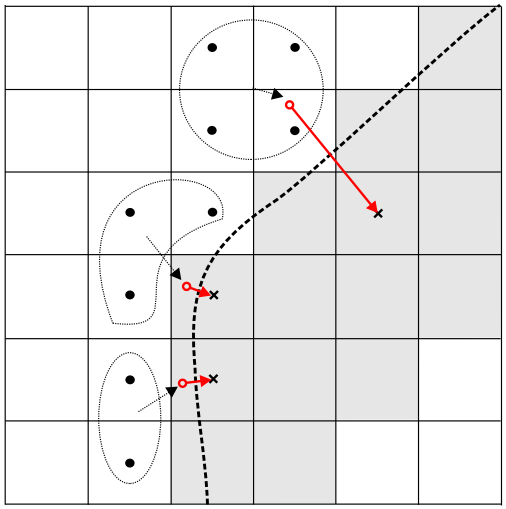

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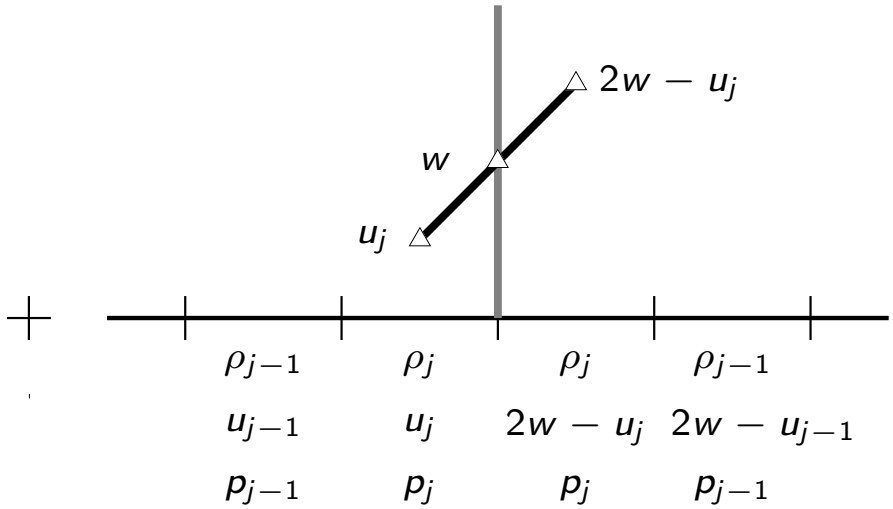
Velocity in ghost cells (slip):

$$\begin{aligned}\mathbf{u}' &= (2\mathbf{w}\cdot\mathbf{n} - \mathbf{u}\cdot\mathbf{n})\mathbf{n} + (\mathbf{u}\cdot\mathbf{t})\mathbf{t} \\ &= 2\left((\mathbf{w}-\mathbf{u})\cdot\mathbf{n}\right)\mathbf{n} + \mathbf{u}\end{aligned}$$



Parallelization

Rigorous domain decomposition

- Data of all levels resides on same node
- Grid hierarchy defines unique "floor-plan"
- Workload estimation

$$\mathcal{W}(\Omega) = \sum_{l=0}^{l_{\max}} \left[\mathcal{N}_l(G_l \cap \Omega) \prod_{\kappa=0}^{l} r_\kappa \right]$$

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  - Synchronization of ghost cells
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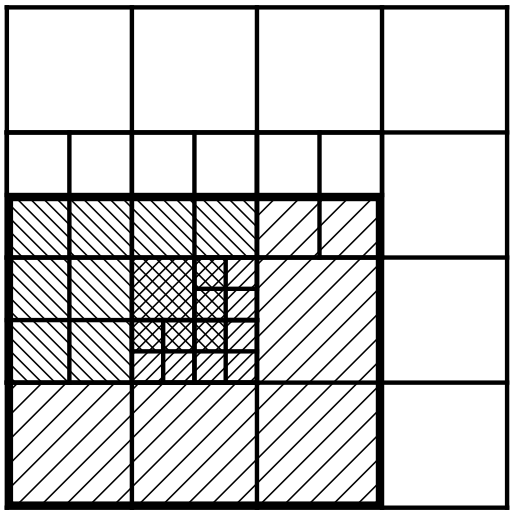

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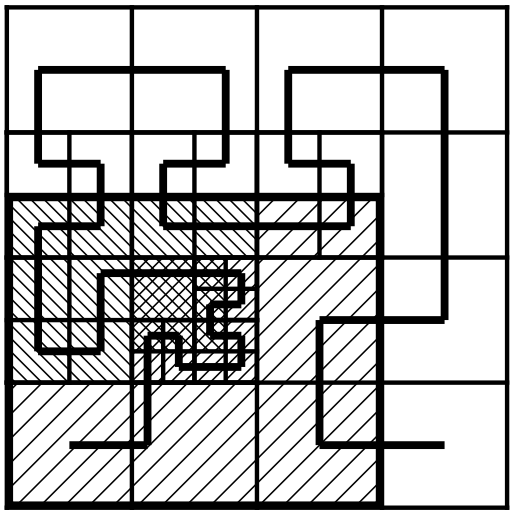

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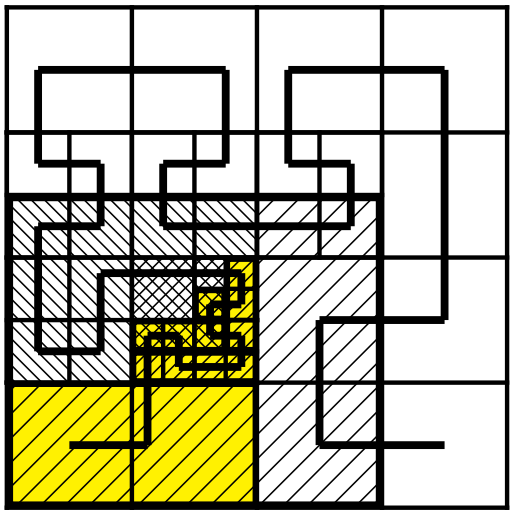

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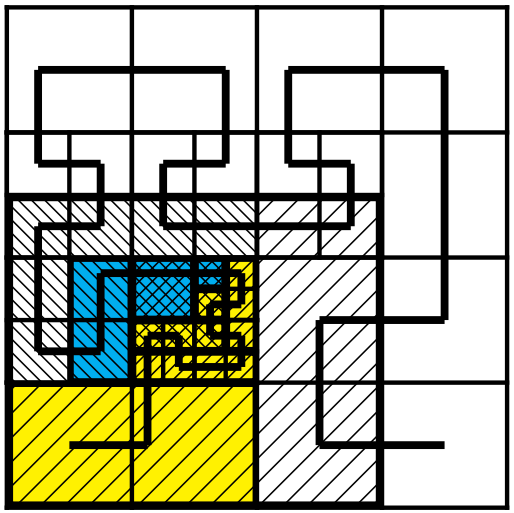

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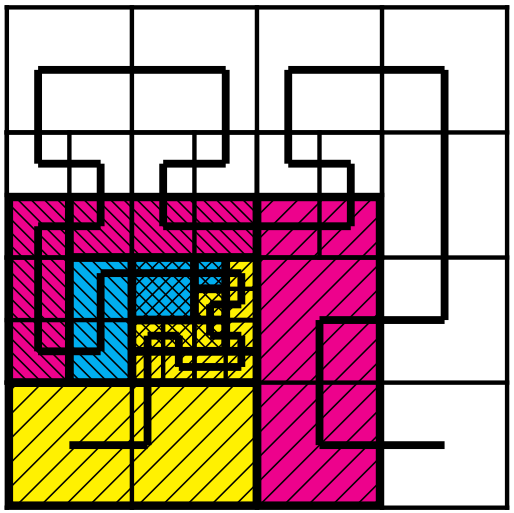

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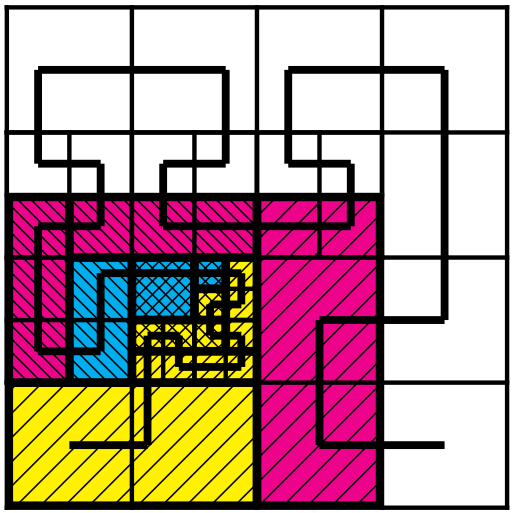


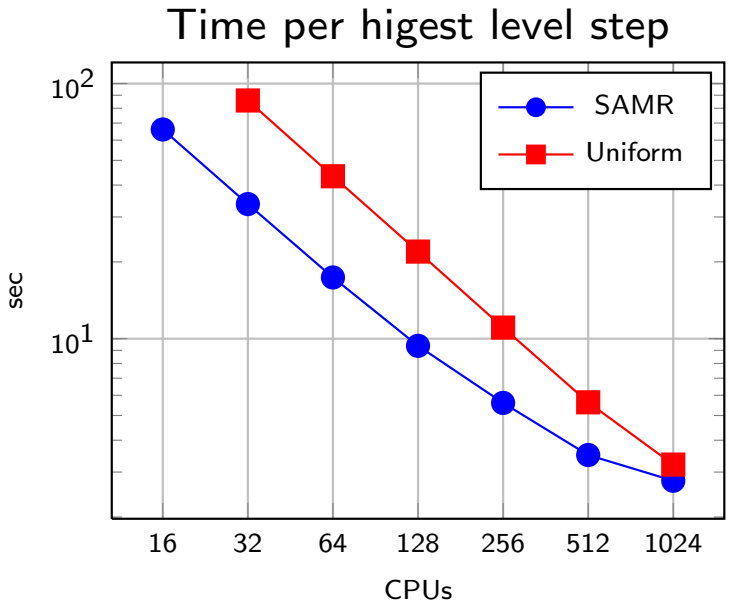
AMROC framework and most important patch solvers

- Implements described algorithms and facilitates easy exchange of the block-based numerical scheme
- Shock-induced combustion with detailed chemistry: [Deiterding, 2003, Deiterding and Bader, 2005, Deiterding, 2011b, Cai et al., 2016, Cai et al., 2018]
- Hybrid WENO methods for LES and DNS: [Pantano et al., 2007, Lombardini and Deiterding, 2010, Ziegler et al., 2011, Cerminara et al., 2018]
- Lattice Boltzmann method for LES: [Fragner and Deiterding, 2016, Feldhusen et al., 2016, Deiterding and Wood, 2016]
- FSI deformation from water hammer: [Cirak et al., 2007, Deiterding et al., 2009a, Perotti et al., 2013, Wan et al., 2017]
- Level-set method for Eulerian solid mechanics: [Barton et al., 2013]
- Ideal magneto-hydrodynamics: [Gomes et al., 2015, Souza Lopes et al., 2018]
- $\sim 500,000$ LOC in C++, C, Fortran-77, Fortran-90
- V2.0 plus FSI coupling routines as open source at http://www.vtf.website
- Used here V3.0 with significantly enhanced parallelization (V2.1 not released)

AMROC strong scalability tests

3D wave propagation method with Roe scheme: spherical blast wave

- Tests run IBM BG/P (mode VN)



$64 \times 32 \times 32$ base grid, 2 additional levels with factors 2, 4; uniform $512 \times 256 \times 256 = 33.6 \cdot 10^6$ cells

Level	Grids	Cells
0	1709	65,536
1	1735	271,048
2	2210	7,190,208

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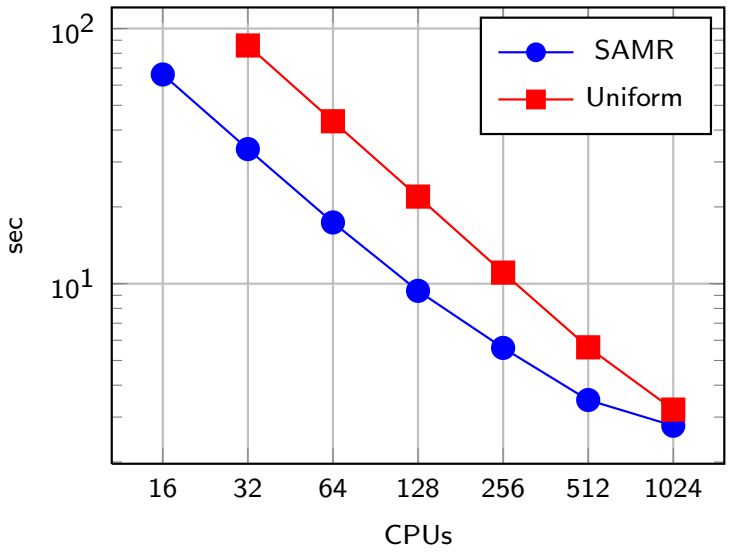

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3D SRT-lattice Boltzmann scheme: flow over rough surface of $19 \times 13 \times 2$ spheres

- Tests run Cray XC30m (Archer)

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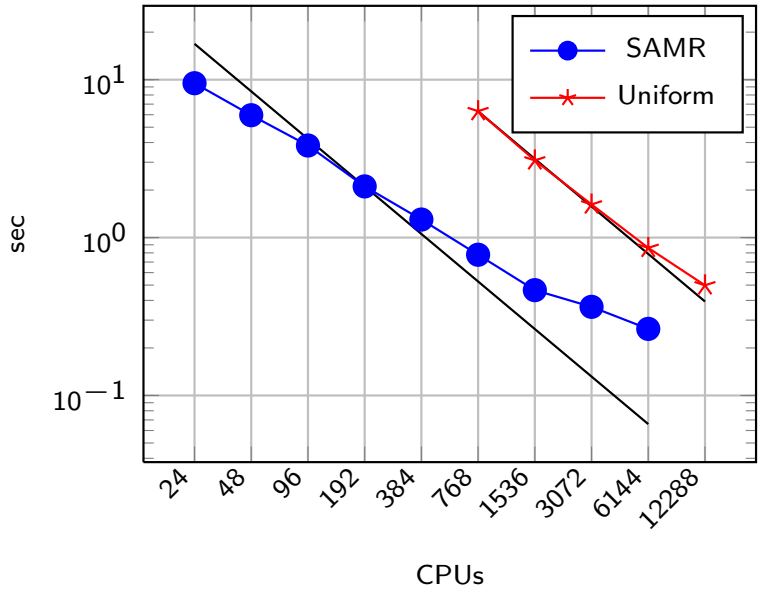
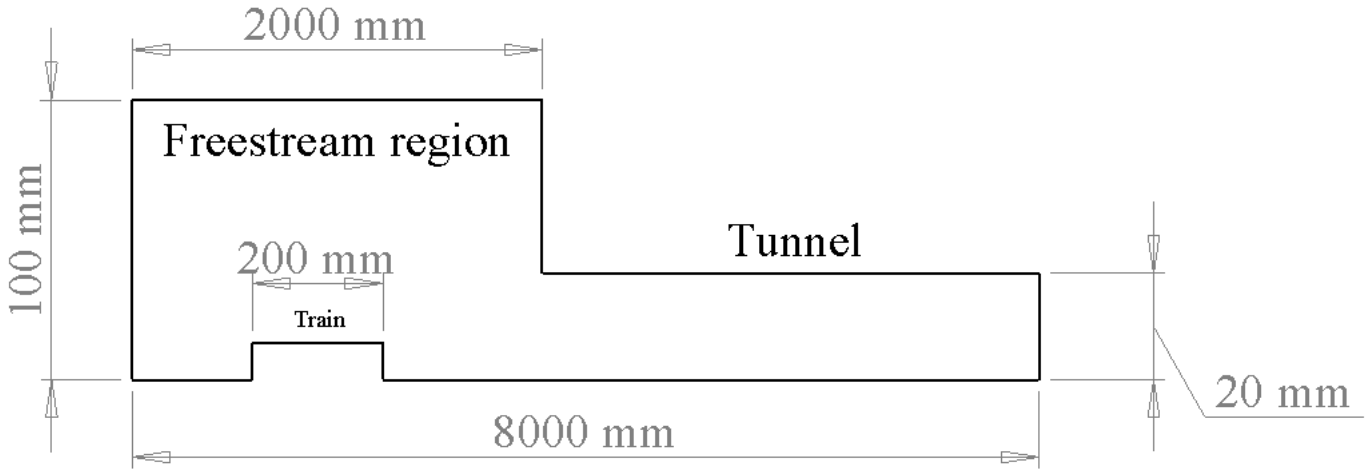


$360 \times 240 \times 108$ base grid, 2 additional levels with factors 2, 4; uniform $1440 \times 1920 \times 432 = 1.19 \cdot 10^9$ cells

Level	Grids	Cells
0	788	9,331,200
1	21367	24,844,504
2	1728	10,838,016

Laboratory tunnel simulator [Zonglin et al., 2002]



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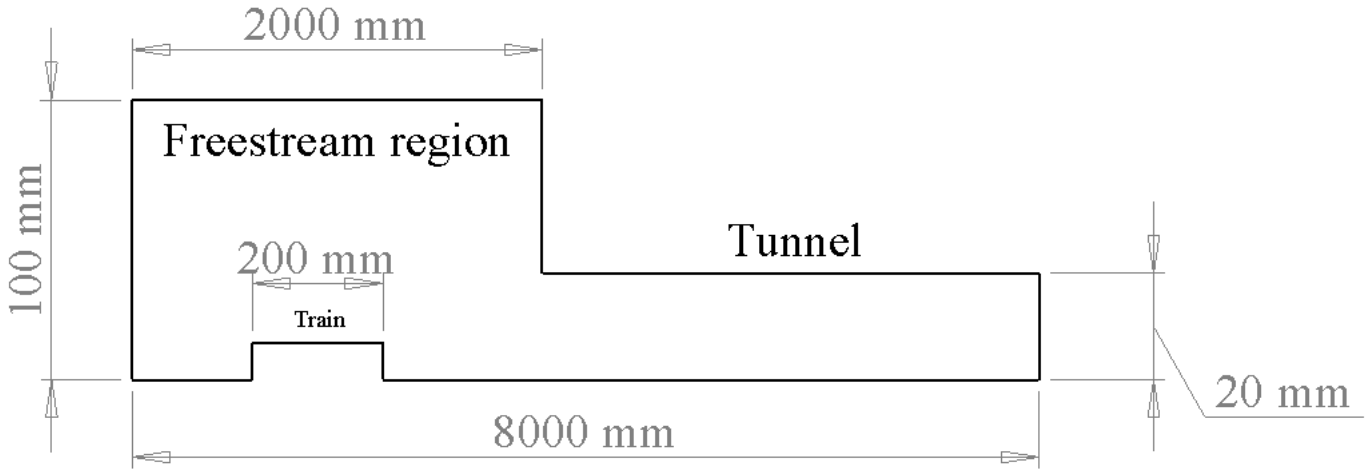

Model solves the inviscid Euler equations

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0$$
$$\partial_t (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) + \nabla p = 0$$
$$\partial_t (\rho E) + \nabla \cdot ((\rho E + p)\mathbf{u}) = 0$$

with $p = (\gamma - 1)(\rho E - \frac{1}{2}\rho \mathbf{u}^T \mathbf{u})$

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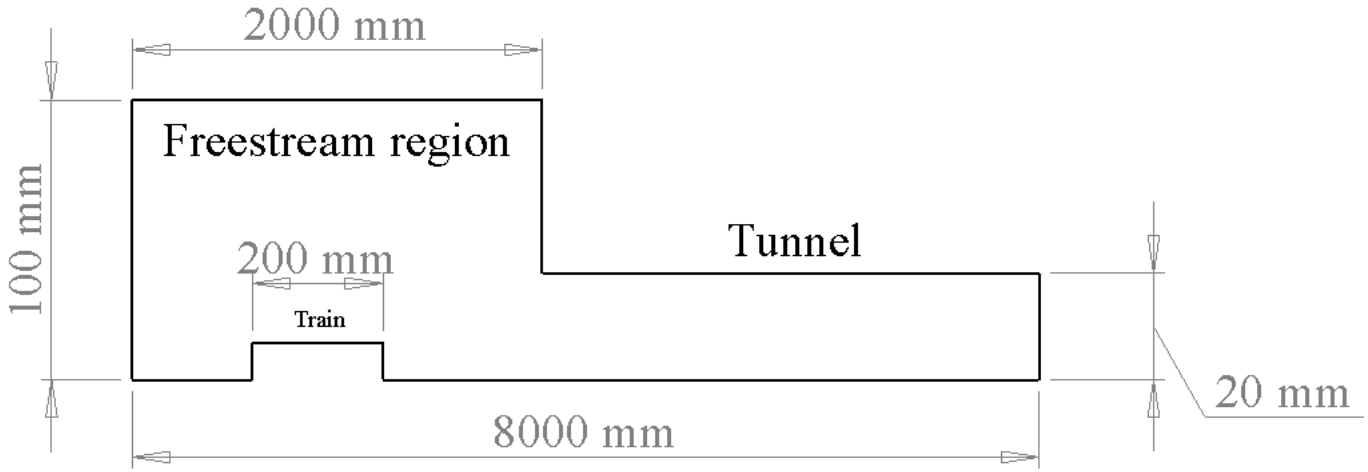


Model solves the inviscid Euler equations

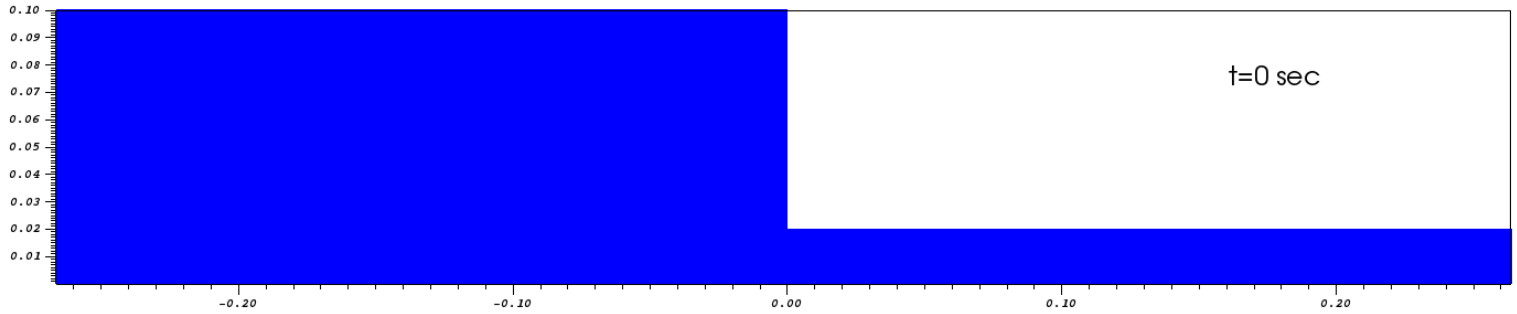
$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0$$
$$\partial_t (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) + \nabla p = 0$$
$$\partial_t (\rho E) + \nabla \cdot ((\rho E + p)\mathbf{u}) = 0$$

with $p = (\gamma - 1)(\rho E - \frac{1}{2}\rho \mathbf{u}^T \mathbf{u})$

- Two-dimensional axi-symmetric computation
- $p_0 = 100\,\mathrm{kPa}$, $\rho_0 = 1.225\,\mathrm{kg/m^3}$, $\gamma = 1.4$
- Roe shock-capturing scheme blended with HLL
- 2nd order accuracy achieved with MUSCL-Hancock method

Basic phenomena – $v_0 = 100\,\mathrm{m/s}$

- 800×25 mesh with Cartesian cut-out $(200, 5)$ to $(800, 25)$
- 2 level of additional refinement by factor 2



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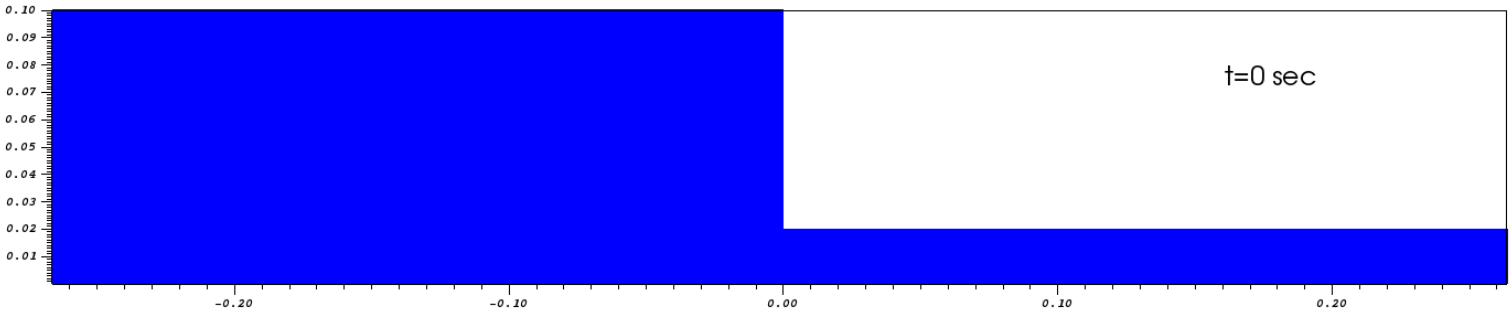
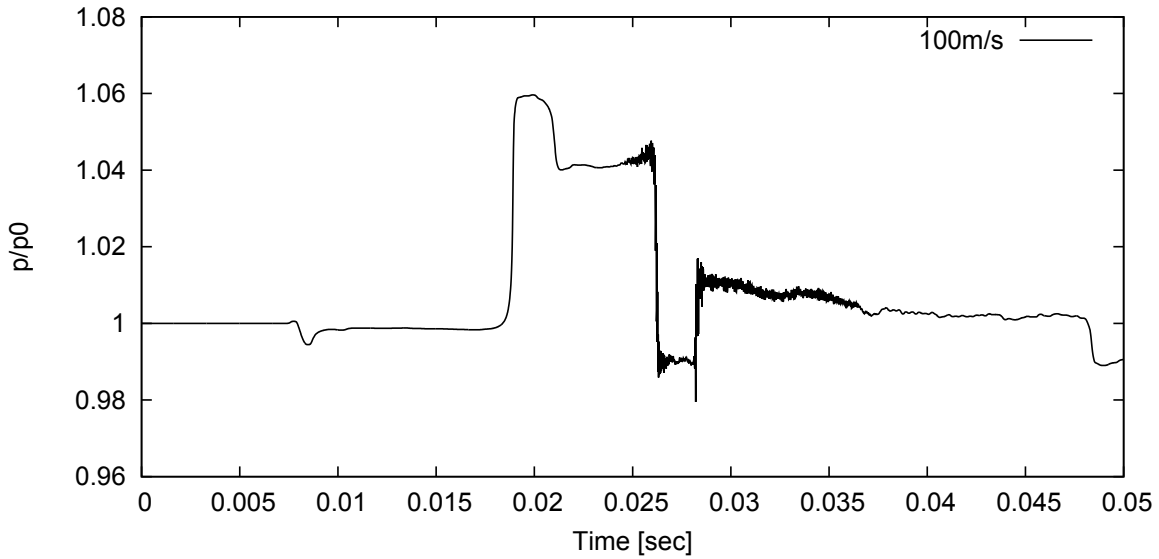
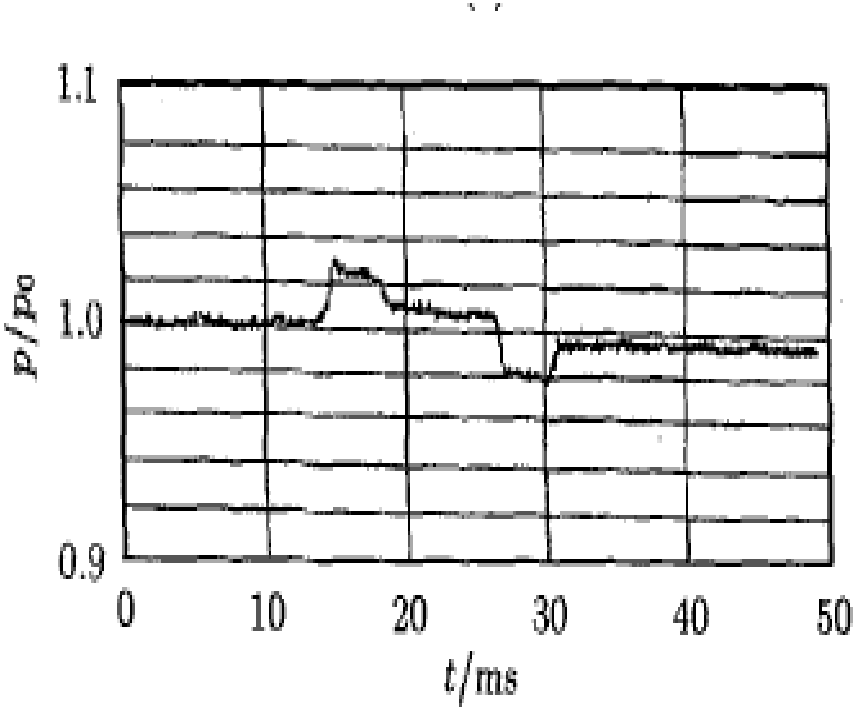
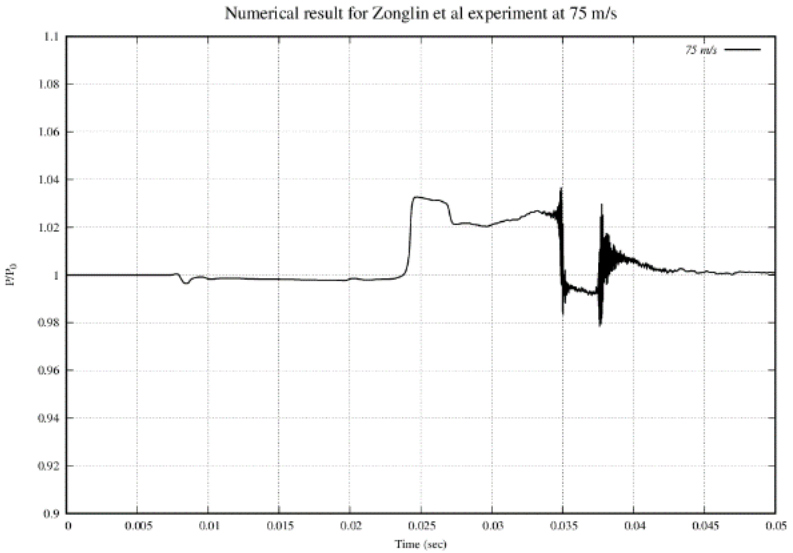




Pressure record at location $(1020\,\mathrm{mm}, 20\,\mathrm{mm})$ inside tunnel

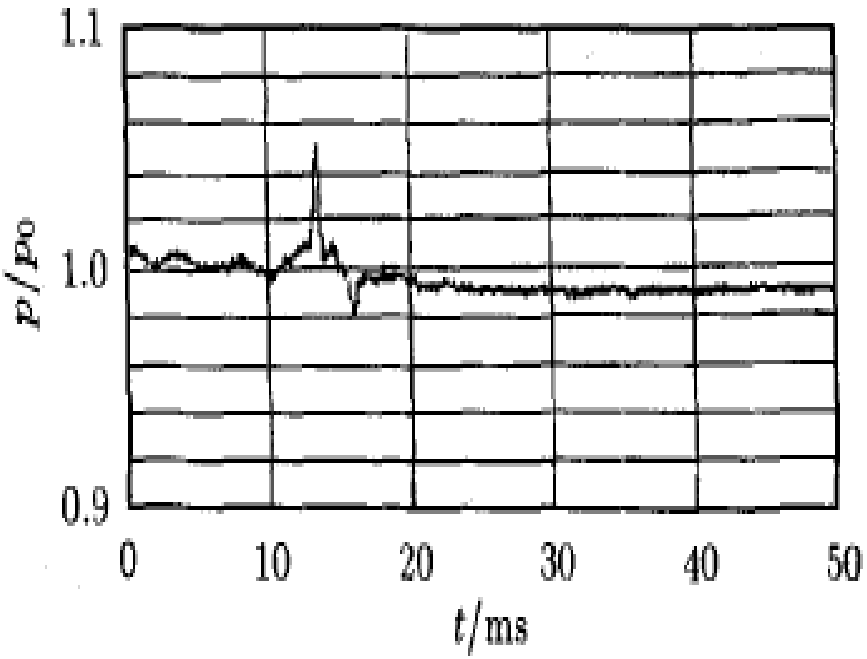
Comparison with experiment – I

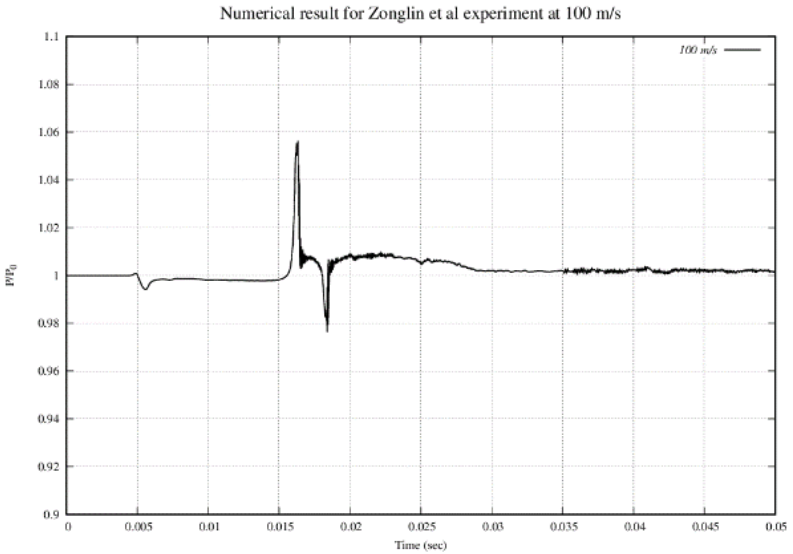




Pressure record at $(1020\,\mathrm{mm}, 20\,\mathrm{mm})$ for $v_0 = 75\,\mathrm{m/s}$. Experiment (left) and AMROC (right)

Comparison with experiment – I

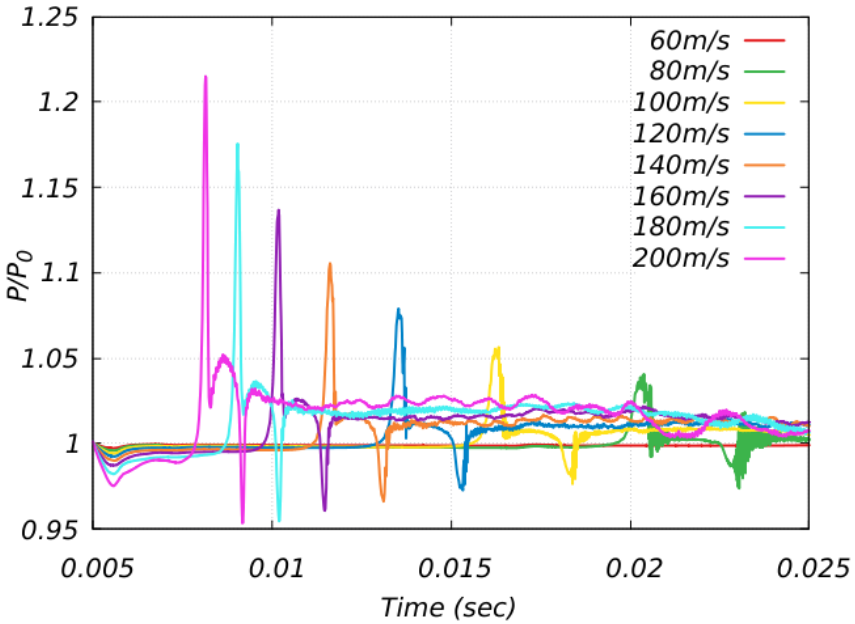




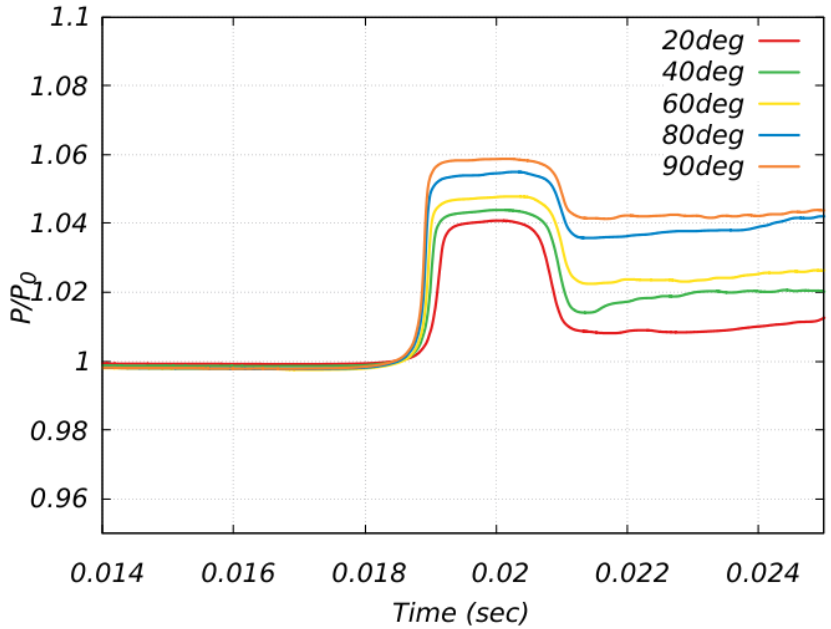
Pressure record at $(40\,\mathrm{mm}, 20\,\mathrm{mm})$ for model velocity $v_0 = 100\,\mathrm{m/s}$. Experiment (left) and AMROC (right)

Variation of velocity and nose half angle

Pressure sensor at $(420\,\mathrm{mm}, 20\,\mathrm{mm})$



Pressure sensor at $(1020\,\mathrm{mm}, 20\,\mathrm{mm})$



- Dependence on v_0^2 is the dynamic pressure influence (left)
- For constant blockage ratio and body velocity, using more pointed noses alleviates the maximal pressure level (right, nose half angle varied)
- For $v_0 \approx 140\,\mathrm{m/s}$ a shock wave (tunnel boom) can be observed. Sharper noses also delay this phenomenon.

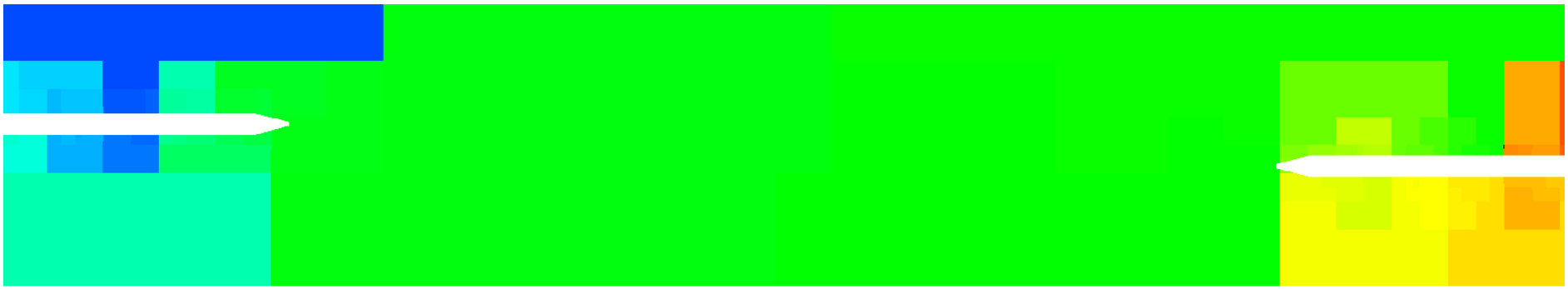
NGT2 prototype setup

- Next Generation Train 2 (NGT2) geometry by the German Aerospace Centre (DLR) [Fragner and Deiterding, 2016, Fragner and Deiterding, 2017]
- Mirrored train head of length $\sim 60\,\mathrm{m}$, no wheels or tracks, train models $0.17\,\mathrm{m}$ above ground above the ground level.
- Train velocities $100\,\mathrm{m/s}$ and $-100\,\mathrm{m/s}$, middle axis $6\,\mathrm{m}$ apart, initial distance between centers $200\,\mathrm{m}$
- Base mesh of $360 \times 40 \times 30$ for domain of $360\,\mathrm{m} \times 40\,\mathrm{m} \times 30\,\mathrm{m}$
- Two/three additional levels, refined by $r_{1,2,3} = 2$. Refinement based on pressure gradient and level set and regenerated at every coarse time step. Parallel redistribution at every level-0 time step.
- On 96 cores Intel Xeon E5-2670 $2.6\,\mathrm{GHz}$ a final $t_e = 3\,\mathrm{sec}$ was reached after $12,385\,\mathrm{sec}$ / $43,395\,\mathrm{sec}$ wall time, i.e., $330\,\mathrm{h}$ and $1157\,\mathrm{h}$ CPU

Passing in open space – AMR and dynamic distribution

Domains of three-level refinement

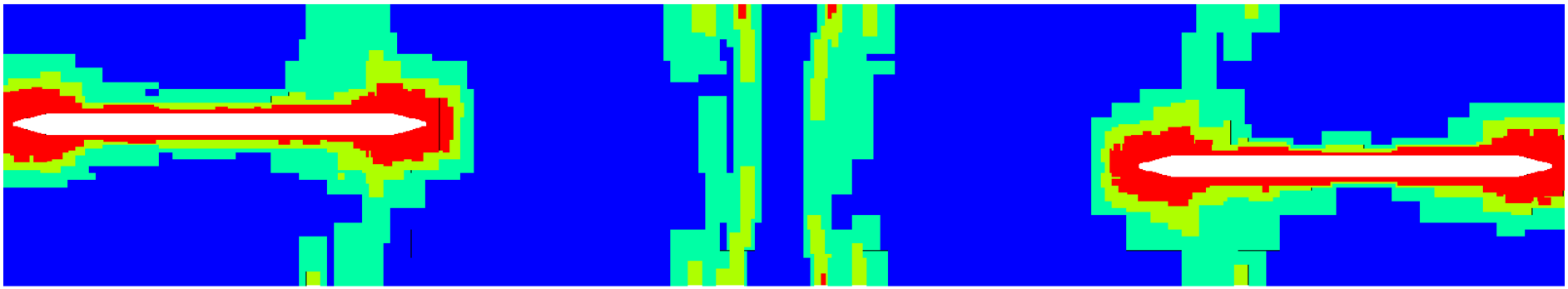
Distribution to 96 processors



Enlargement of domain center shown

Passing in open space – AMR and dynamic distribution

Domains of three-level refinement



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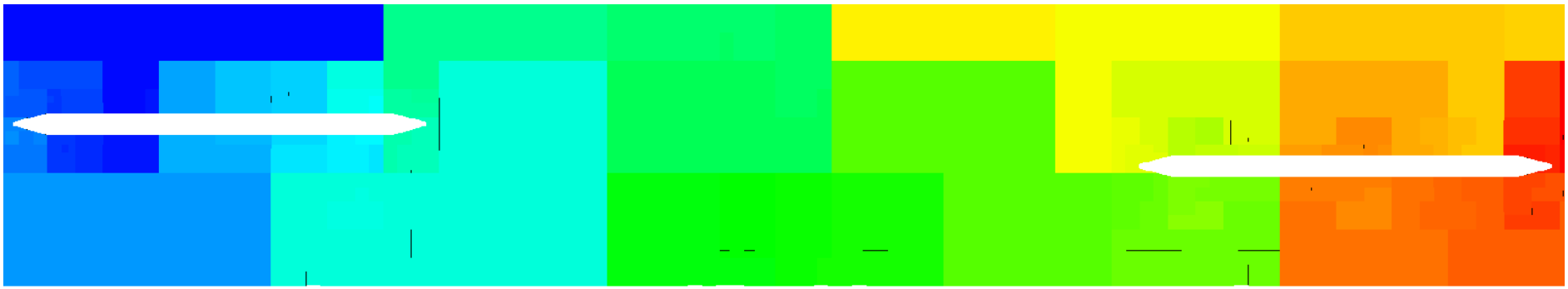

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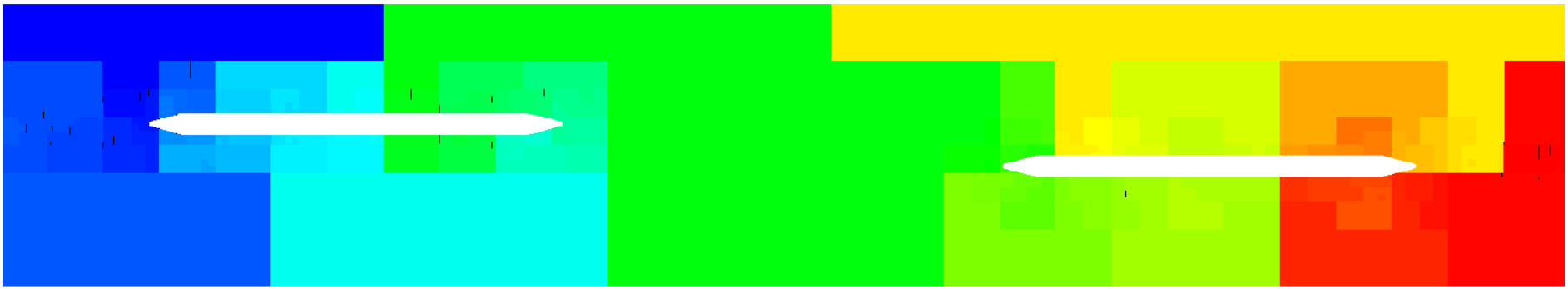
Passing in open space – AMR and dynamic distribution

Domains of three-level refinement

Distribution to 96 processors



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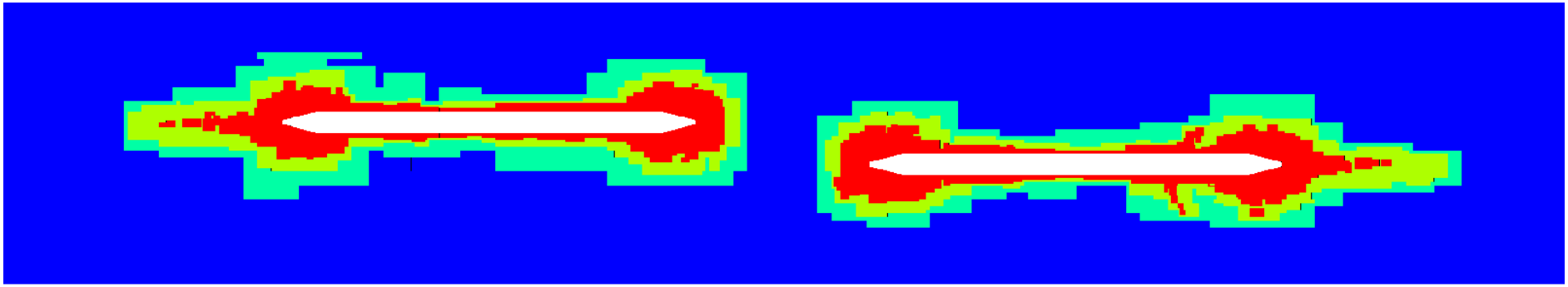
Passing in open space – AMR and dynamic distribution

Domains of three-level refinement



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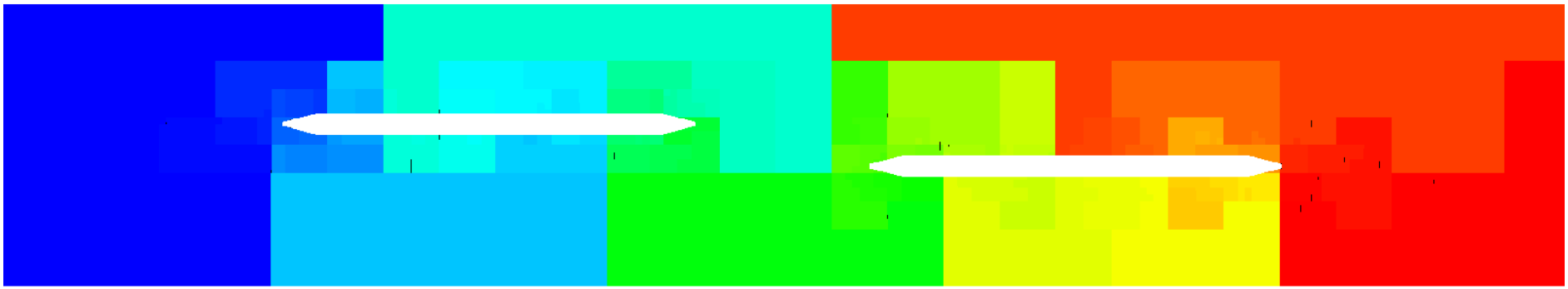

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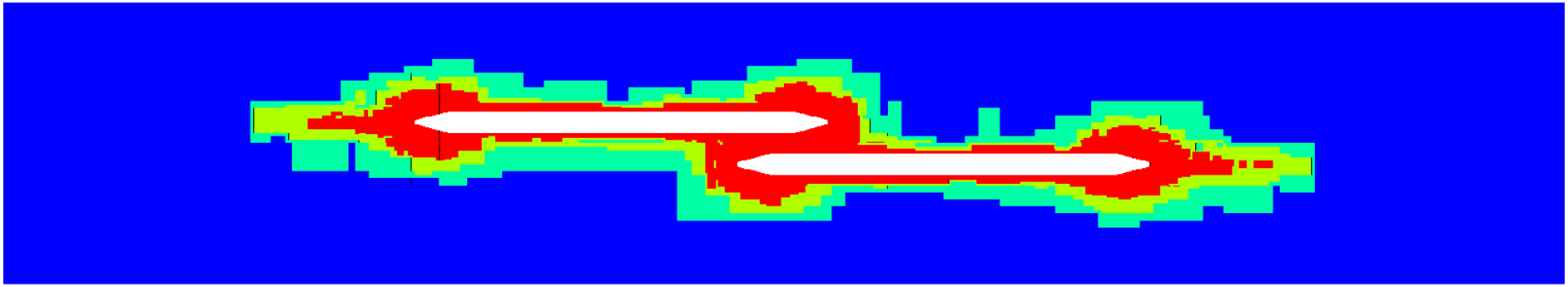
Passing in open space – AMR and dynamic distribution

Domains of three-level refinement



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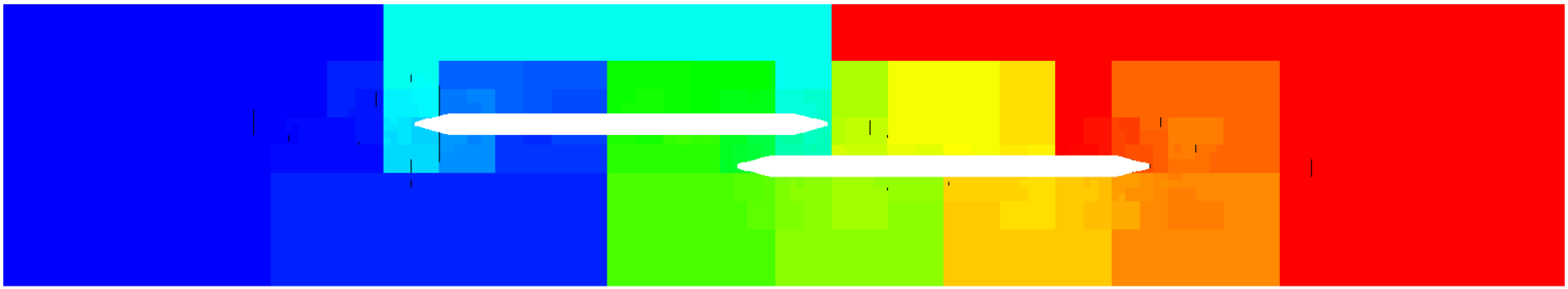

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Passing in open space – AMR and dynamic distribution

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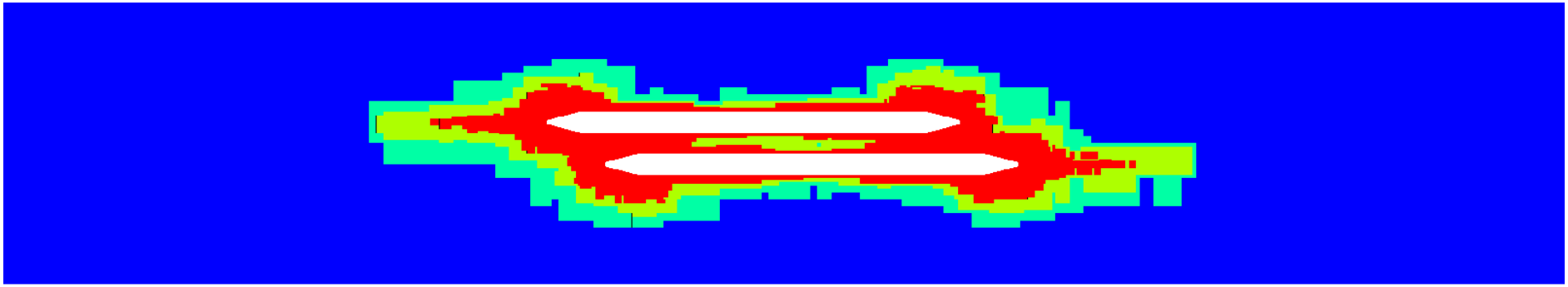

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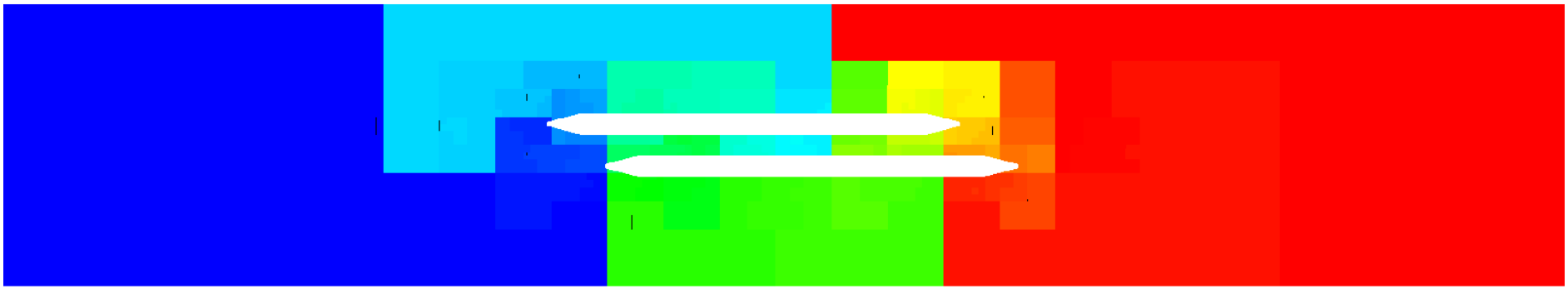
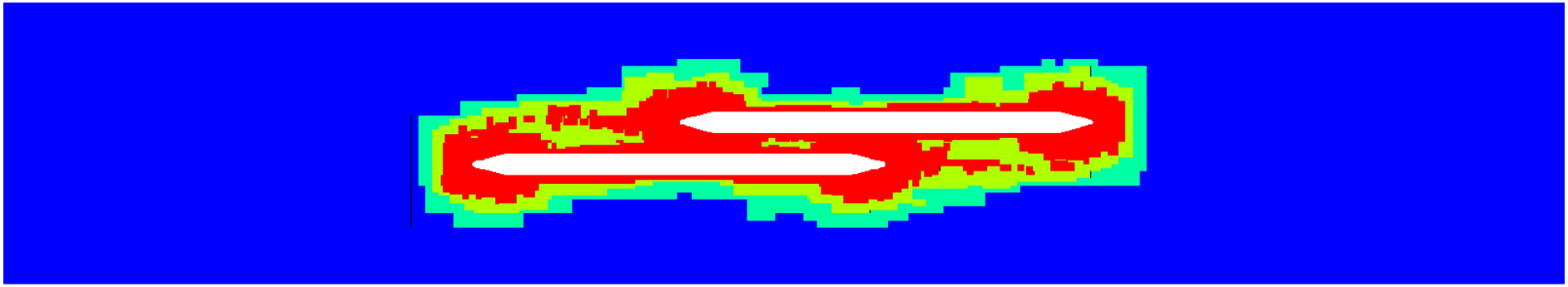


Enlargement of domain center shown

Passing in open space – AMR and dynamic distribution

Domains of three-level refinement

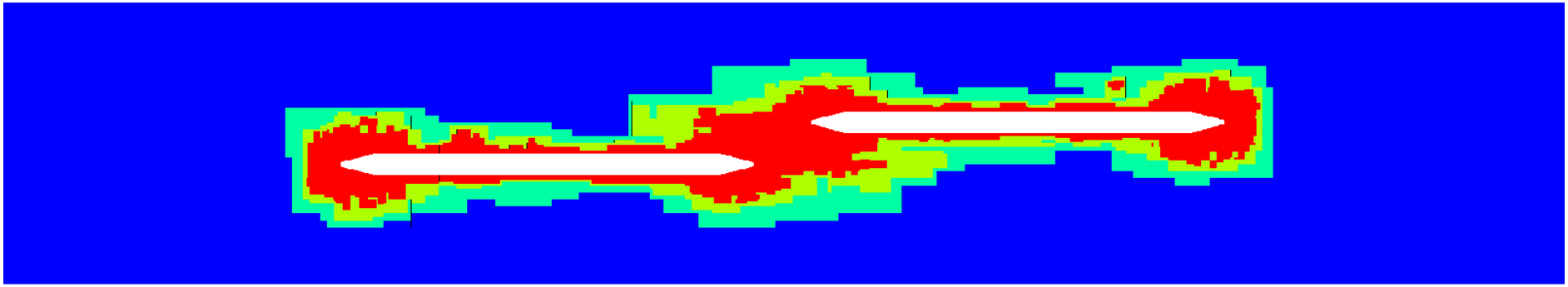


Distribution to 96 processors

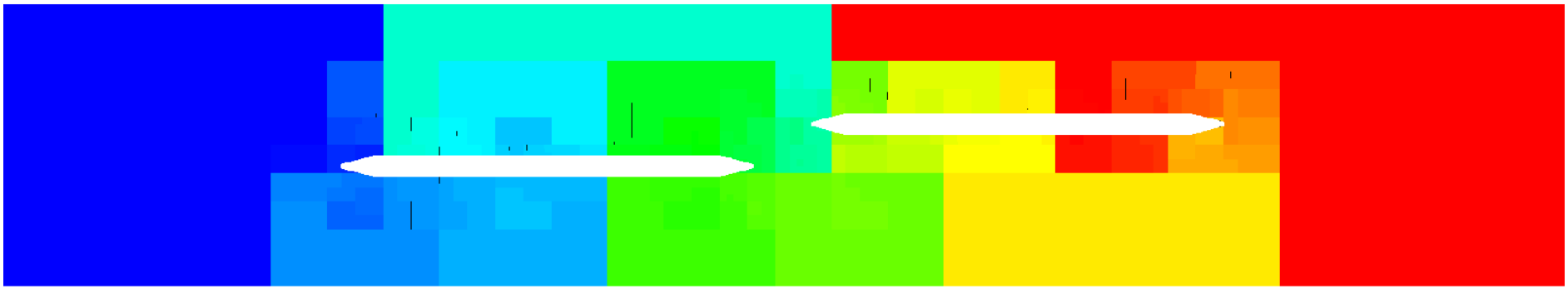
Enlargement of domain center shown

Passing in open space – AMR and dynamic distribution

Domains of three-level refinement



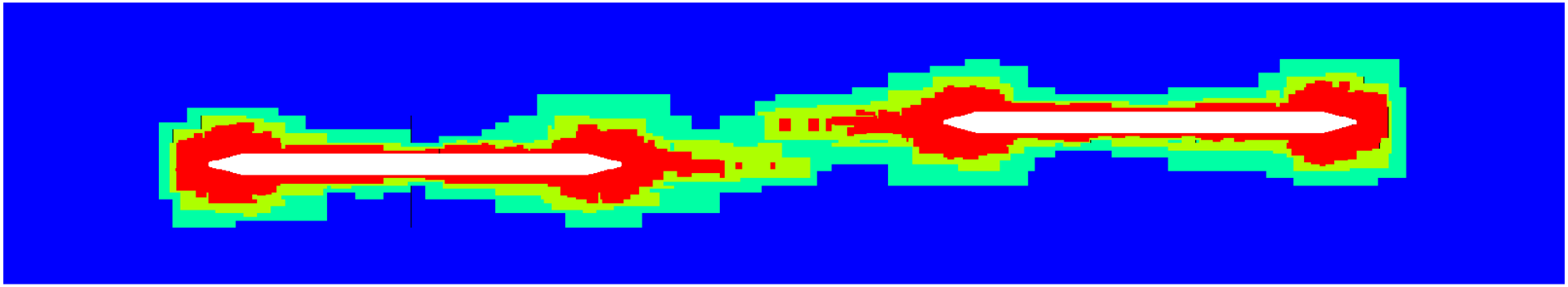
Distribution to 96 processors



Enlargement of domain center shown

Passing in open space – AMR and dynamic distribution

Domains of three-level refinement



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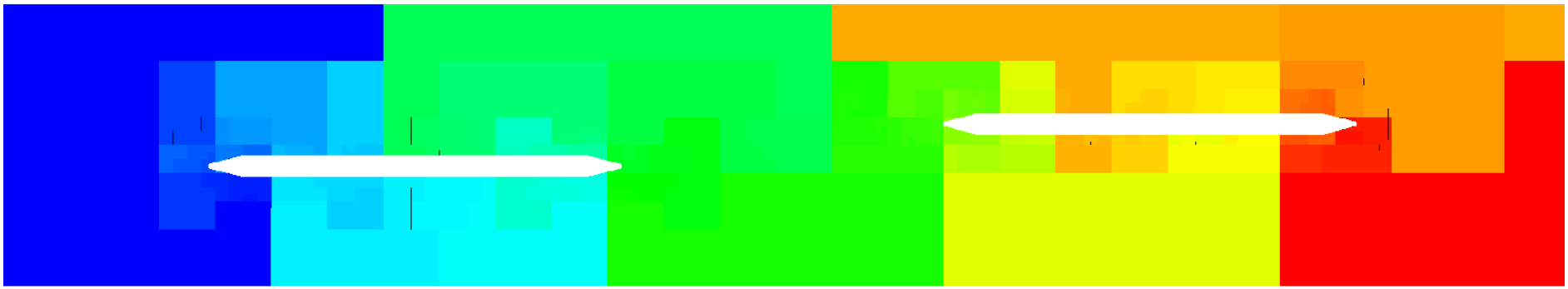
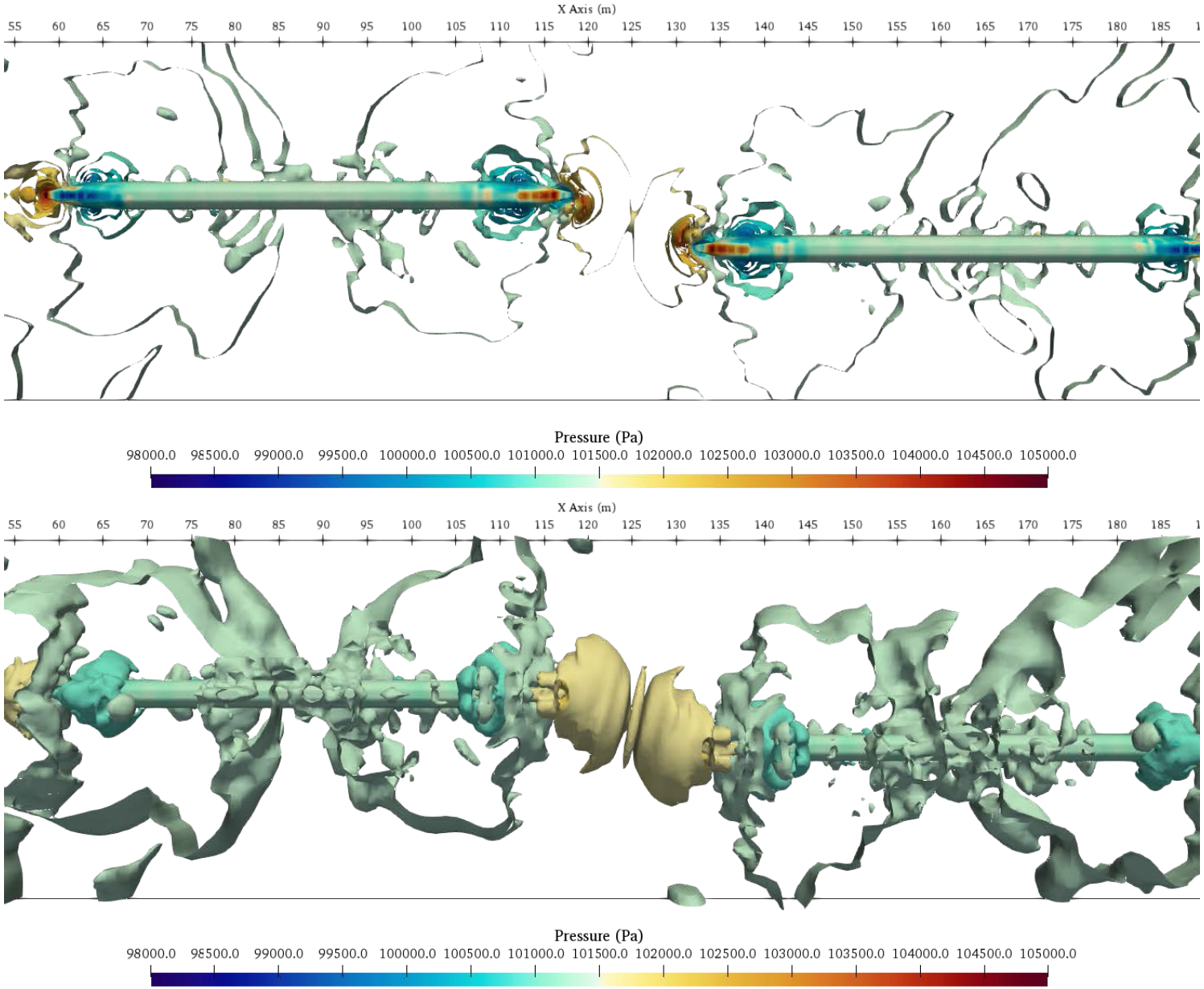
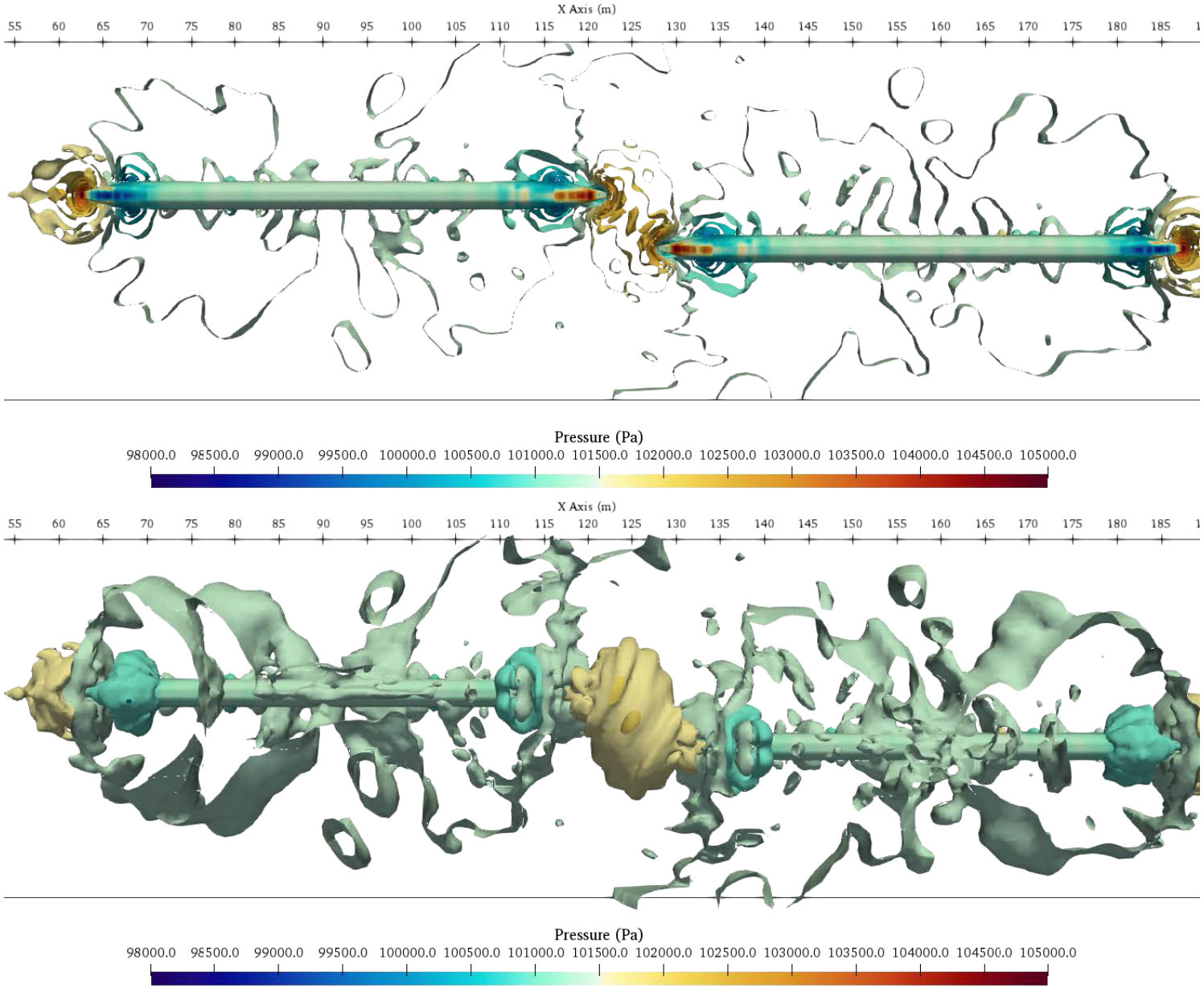


Enlargement of domain center shown

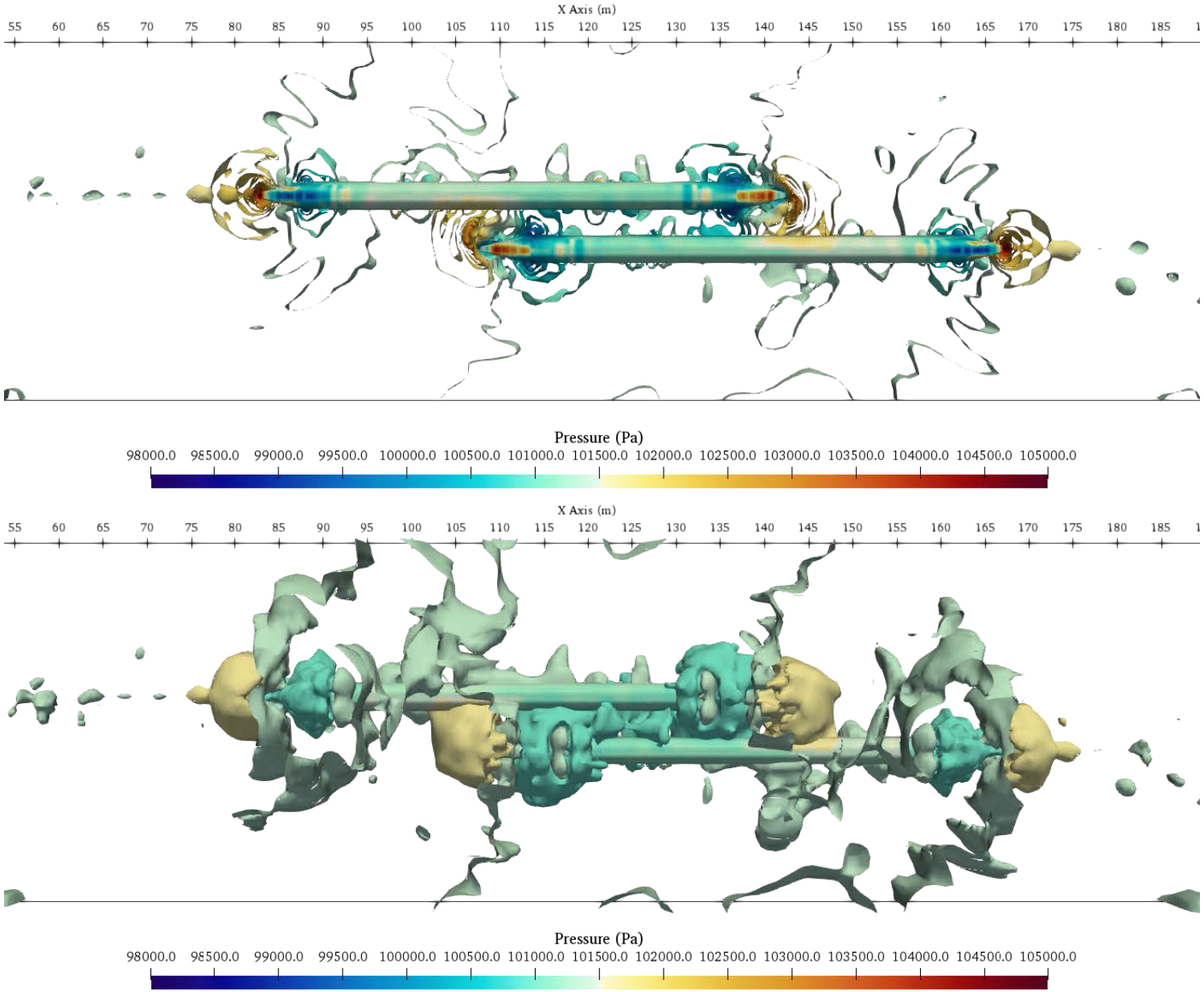
Pressure isosurfaces



Pressure isosurfaces



Pressure isosurfaces



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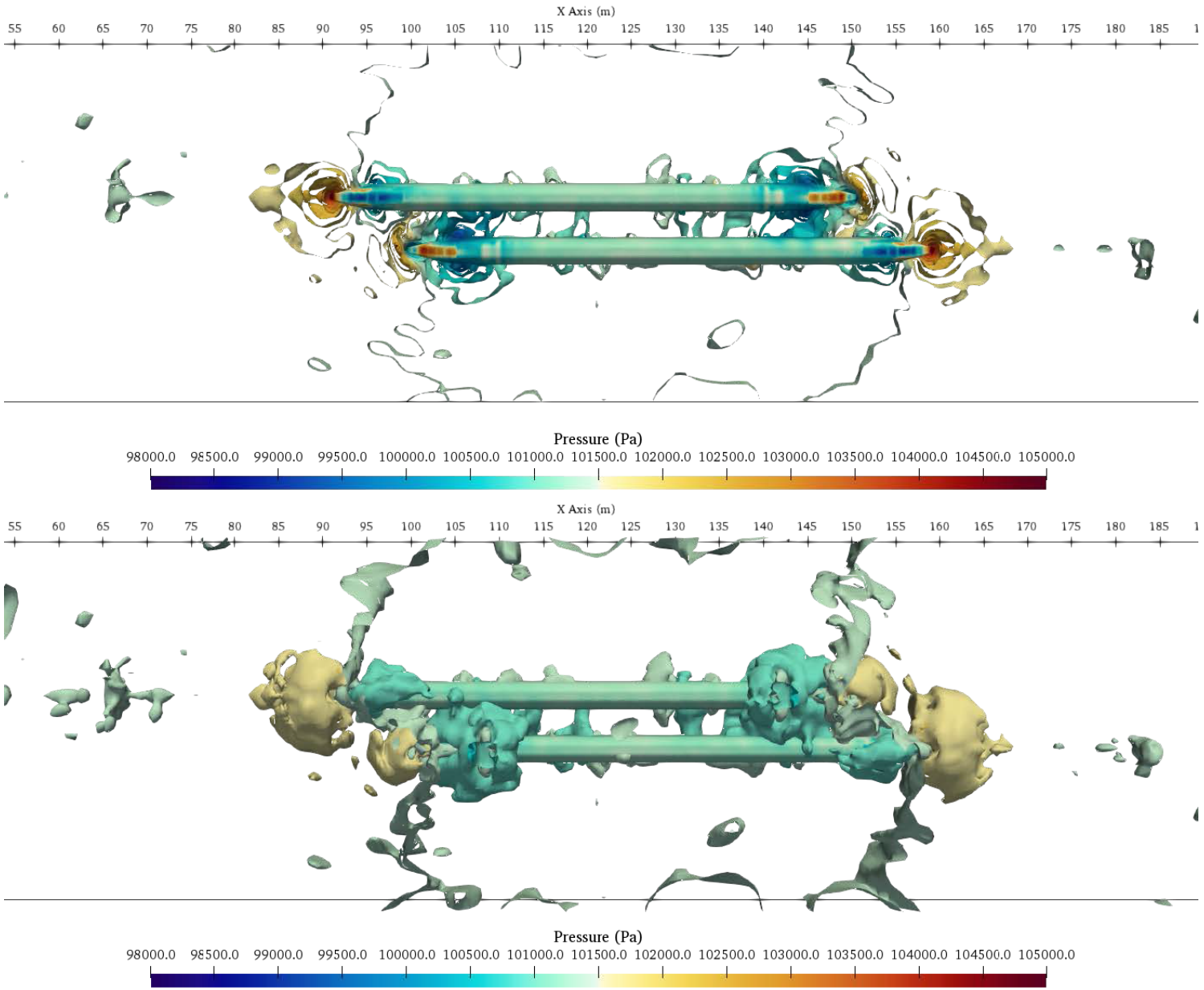
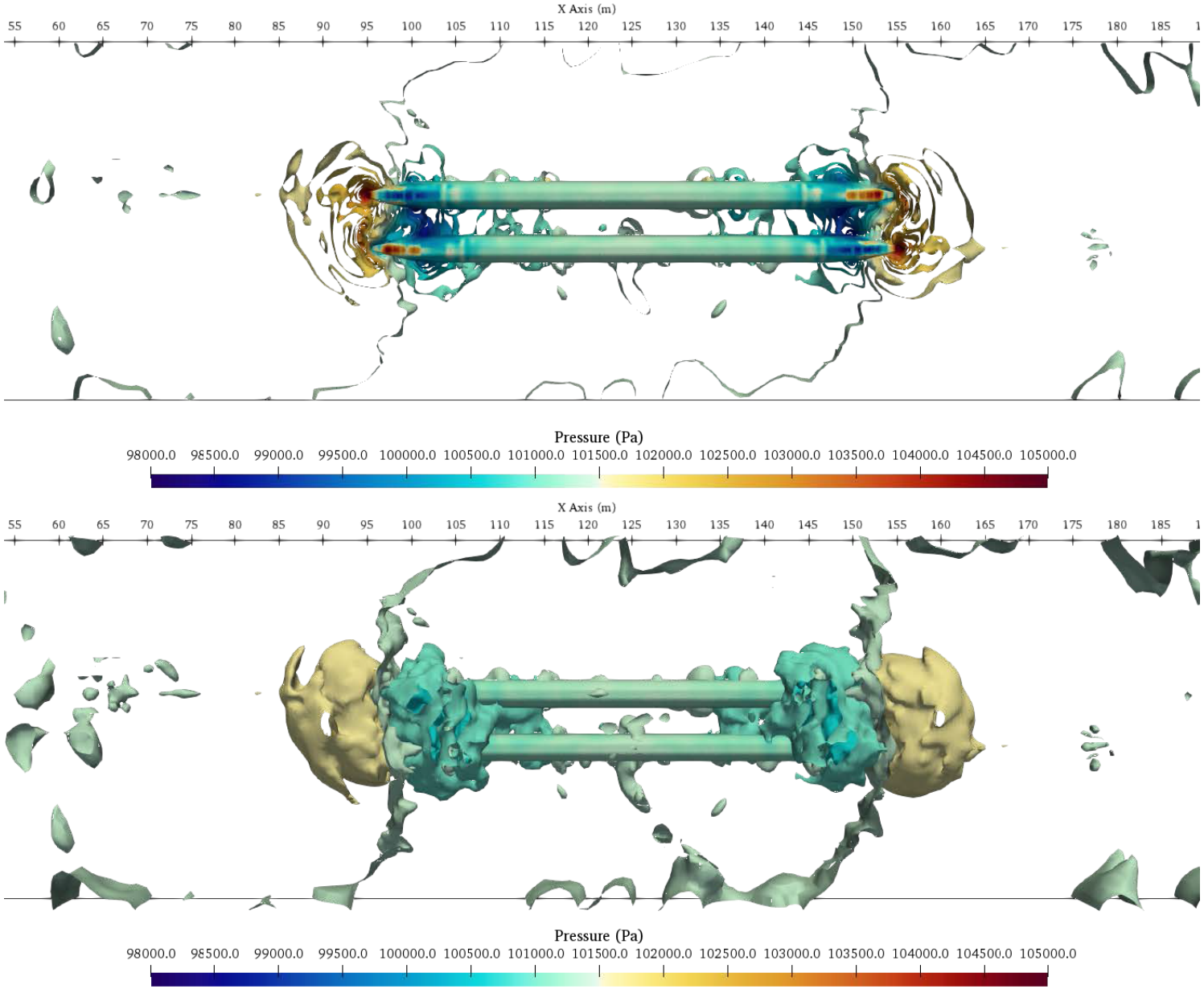


Pressure isosurfaces



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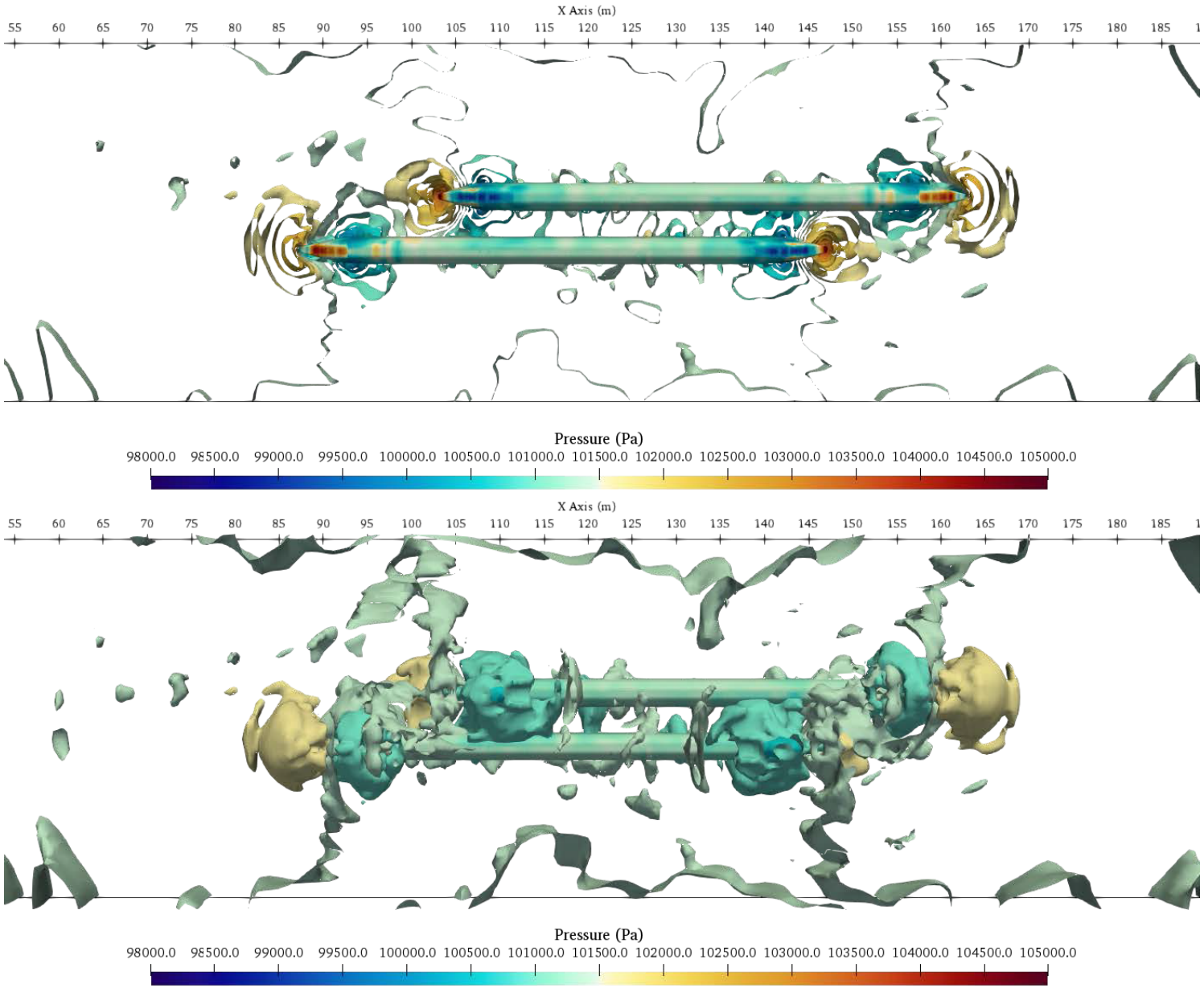

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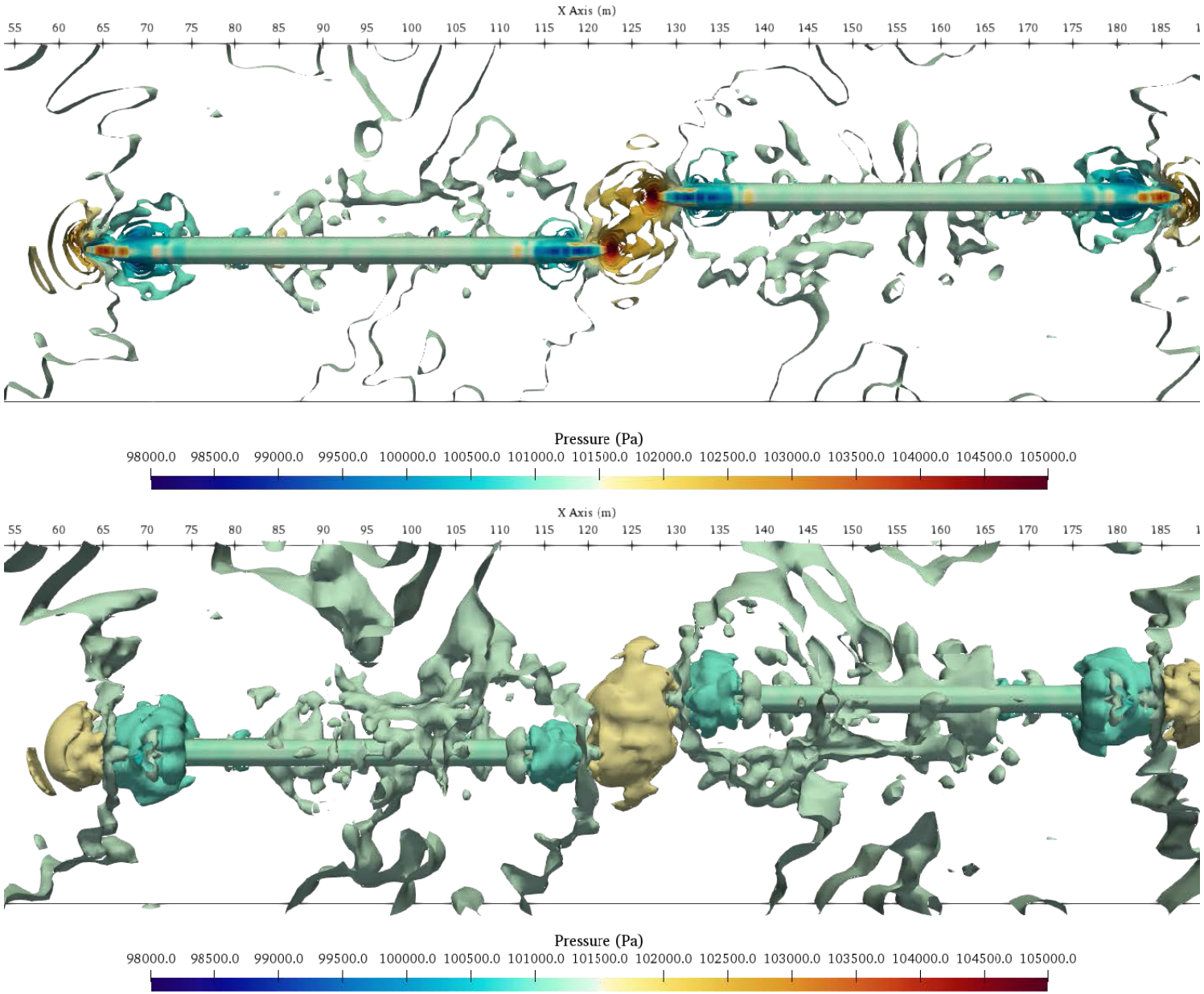

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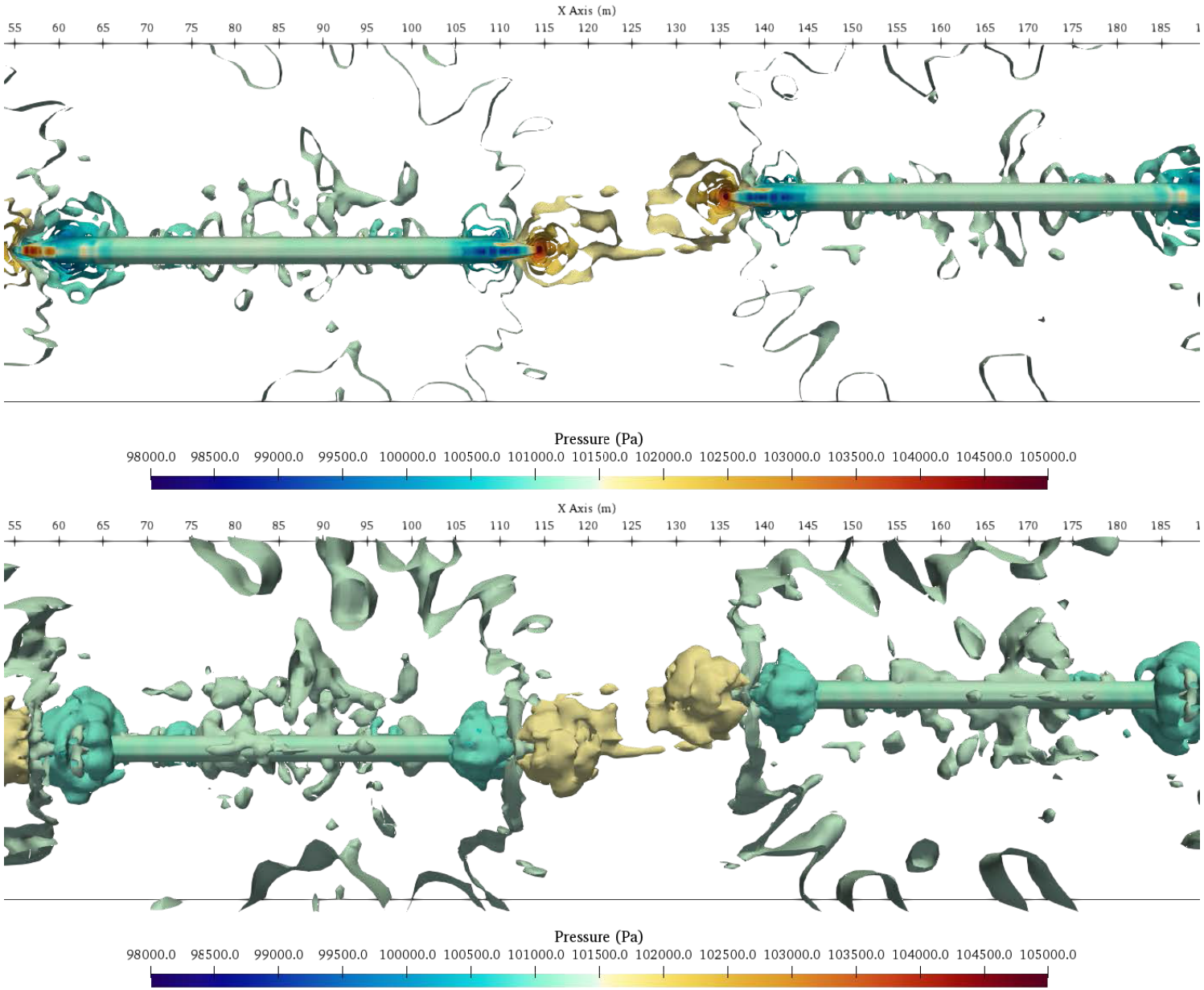
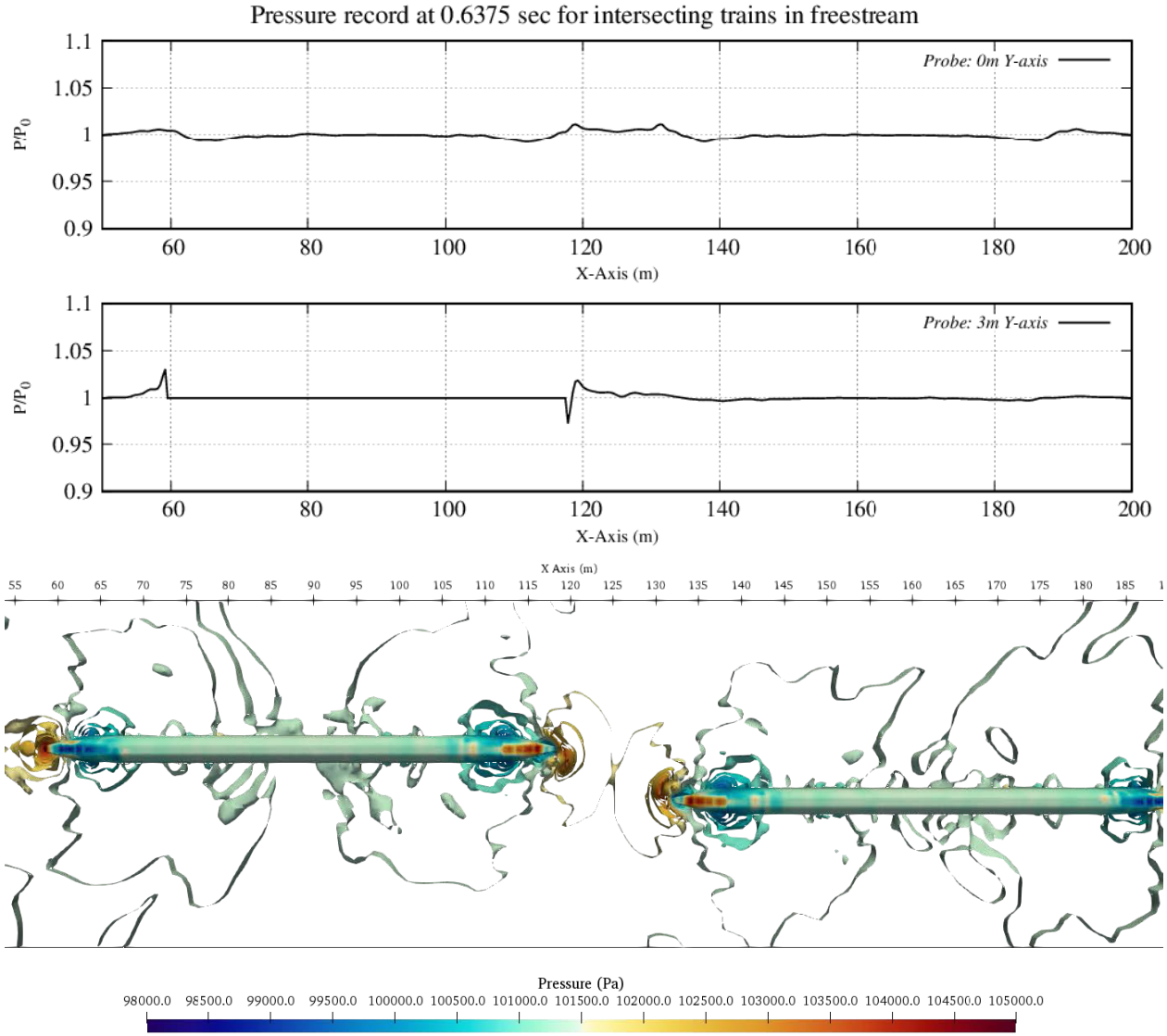


Pressure transects



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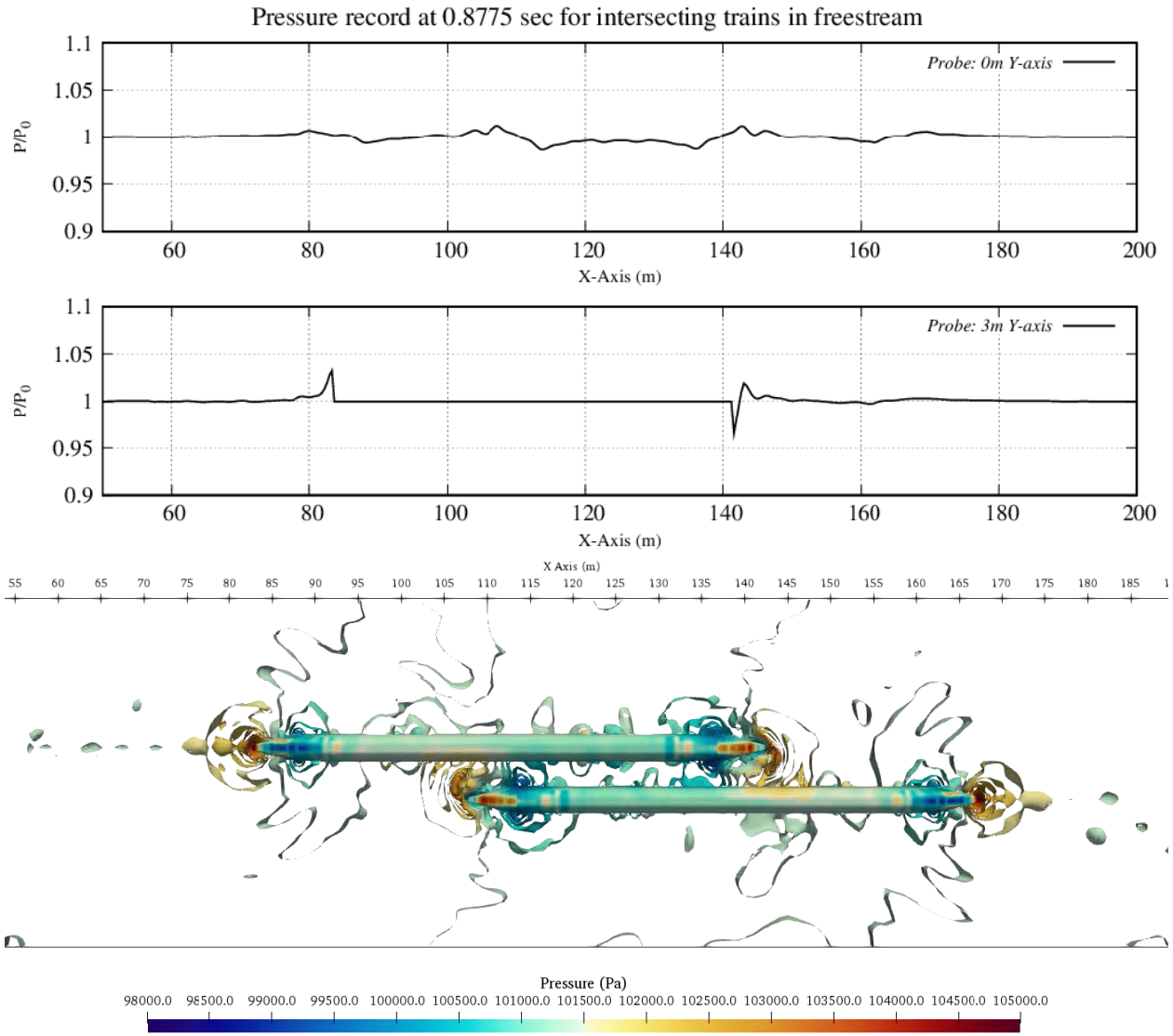

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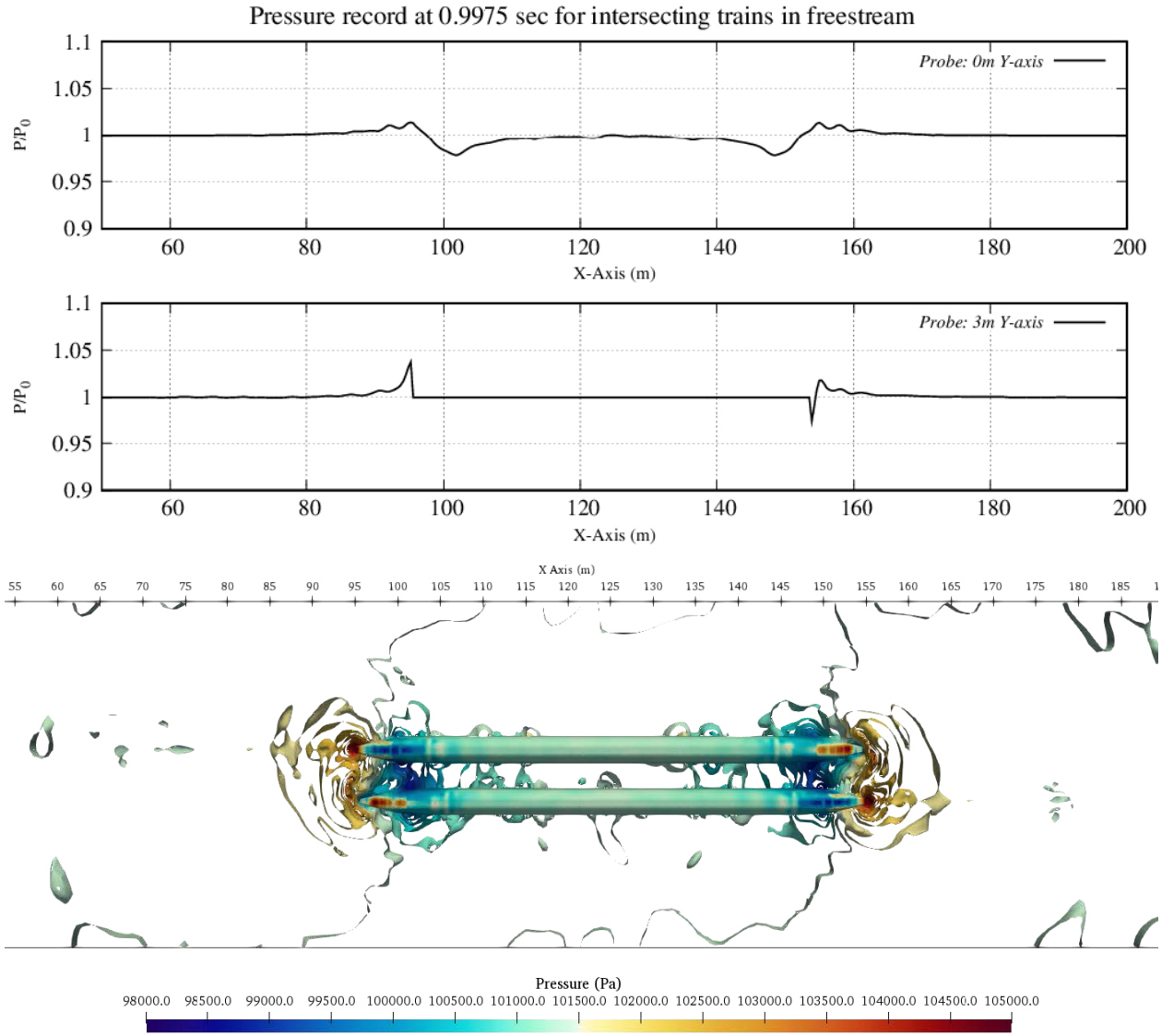

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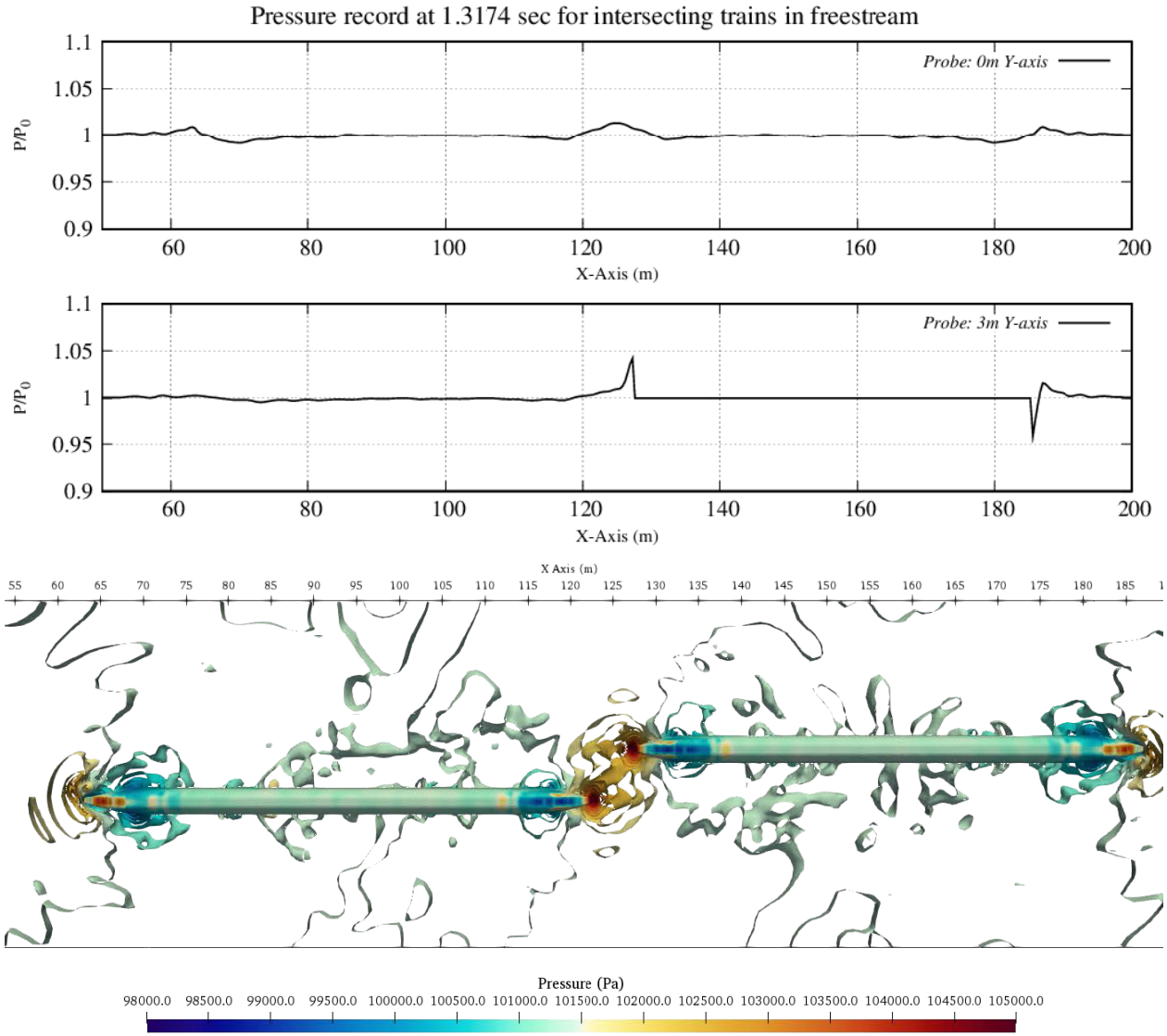
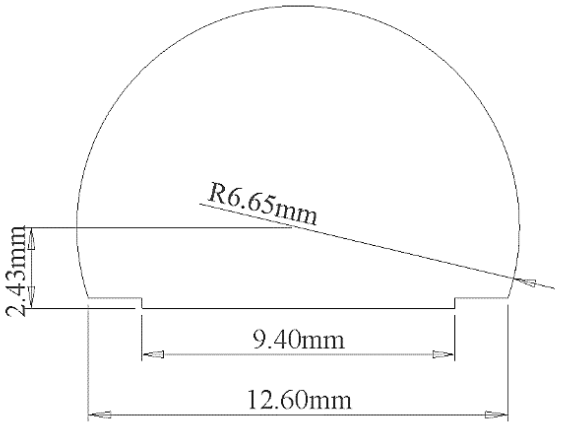


Setup with realistic tunnel shape

- Two NGT2 trains again at velocities $100\,\mathrm{m/s}$ and $-100\,\mathrm{m/s}$
- Prototype straight double track tunnel of $640\,\mathrm{m}$ length, initial distance between centers of trains $820\,\mathrm{m}$
- Base mesh of $1060 \times 36 \times 24$ for domain of $1060\,\mathrm{m} \times 36\,\mathrm{m} \times 24\,\mathrm{m}$, three levels refined by $r_{1,2,3} = 2$
- On 96 cores Intel Xeon E5-2670 $2.6\,\mathrm{GHz}$ a final $t_e = 5\,\mathrm{sec}$ was reached after $84,651\,\mathrm{sec}$ wall time, i.e., $2257\,\mathrm{h}$ CPU



Tunnel shape

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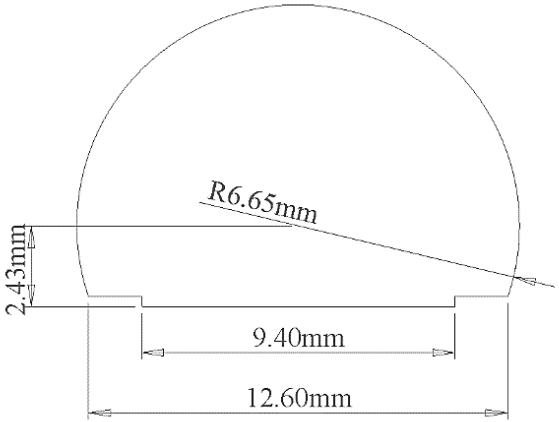
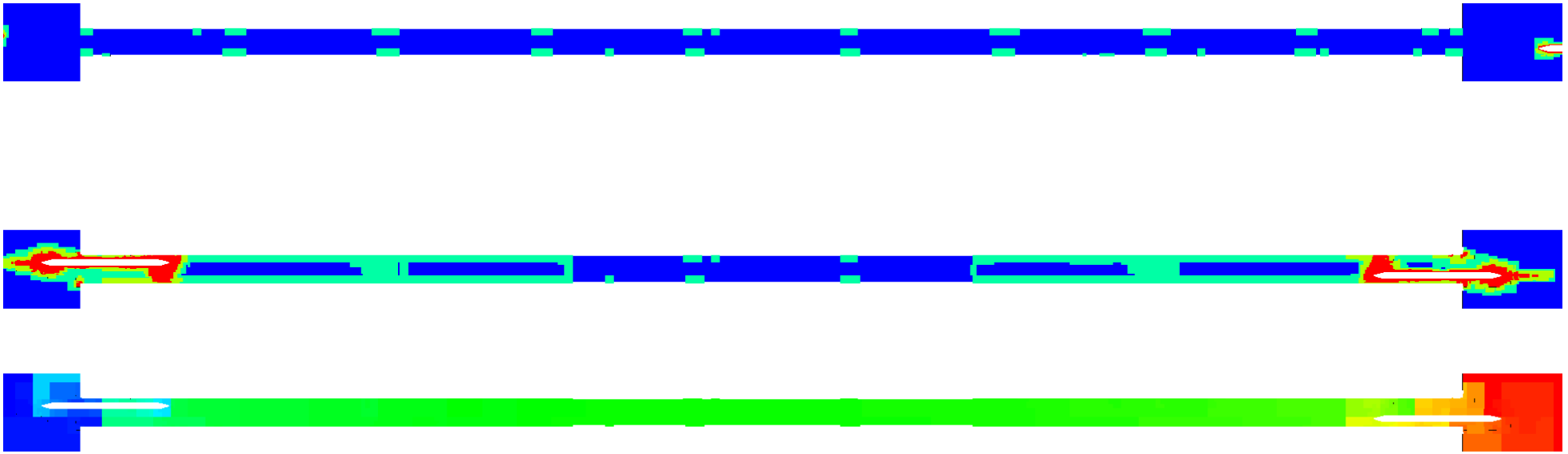


Tunnel shape



Setup with realistic tunnel shape

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- Prototype straight double track tunnel of $640\,\mathrm{m}$ length, initial distance between centers of trains $820\,\mathrm{m}$
- Base mesh of $1060 \times 36 \times 24$ for domain of $1060\,\mathrm{m} \times 36\,\mathrm{m} \times 24\,\mathrm{m}$, three levels refined by $r_{1,2,3} = 2$
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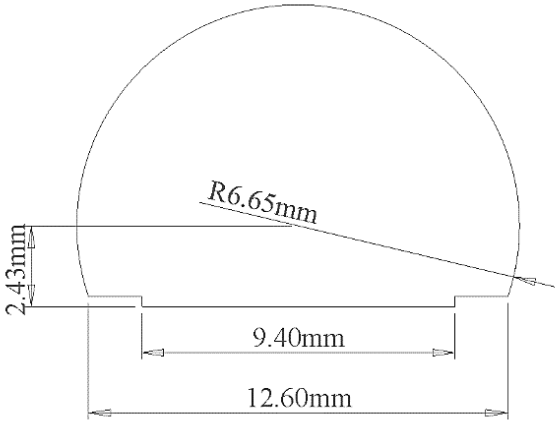
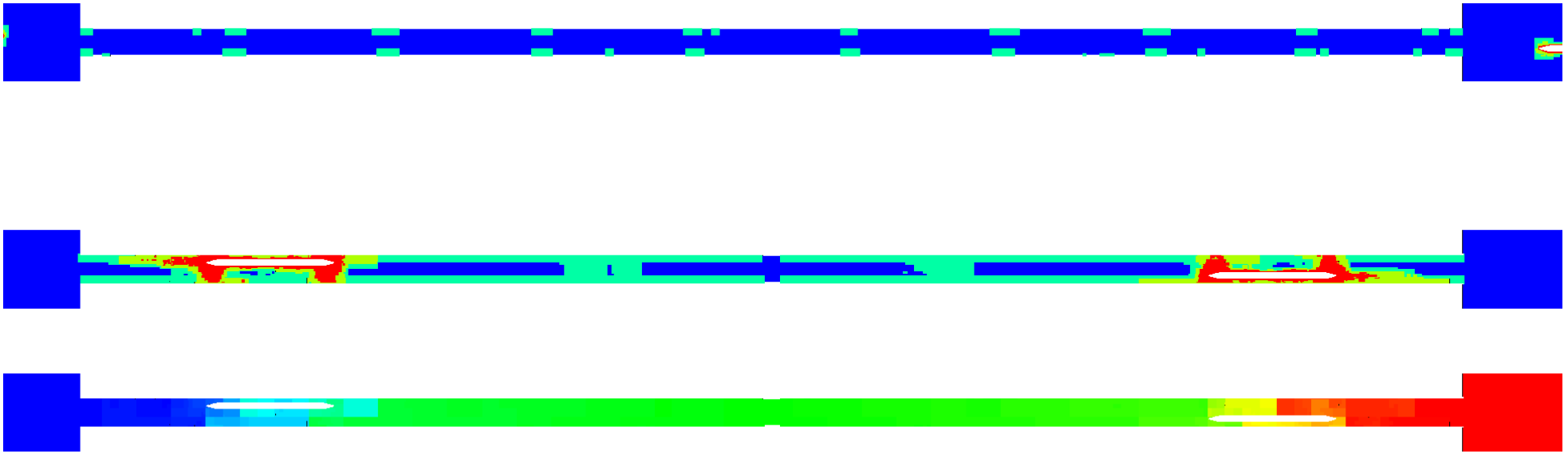


Tunnel shape



Setup with realistic tunnel shape

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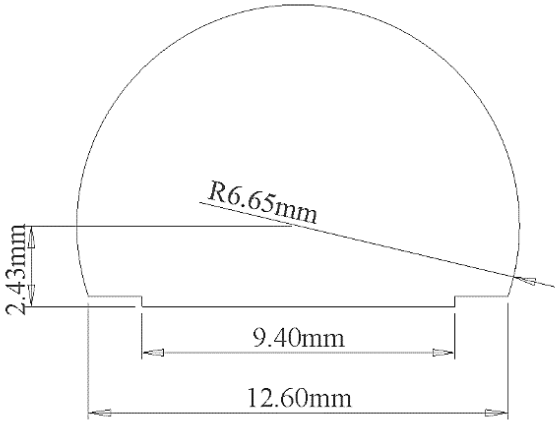
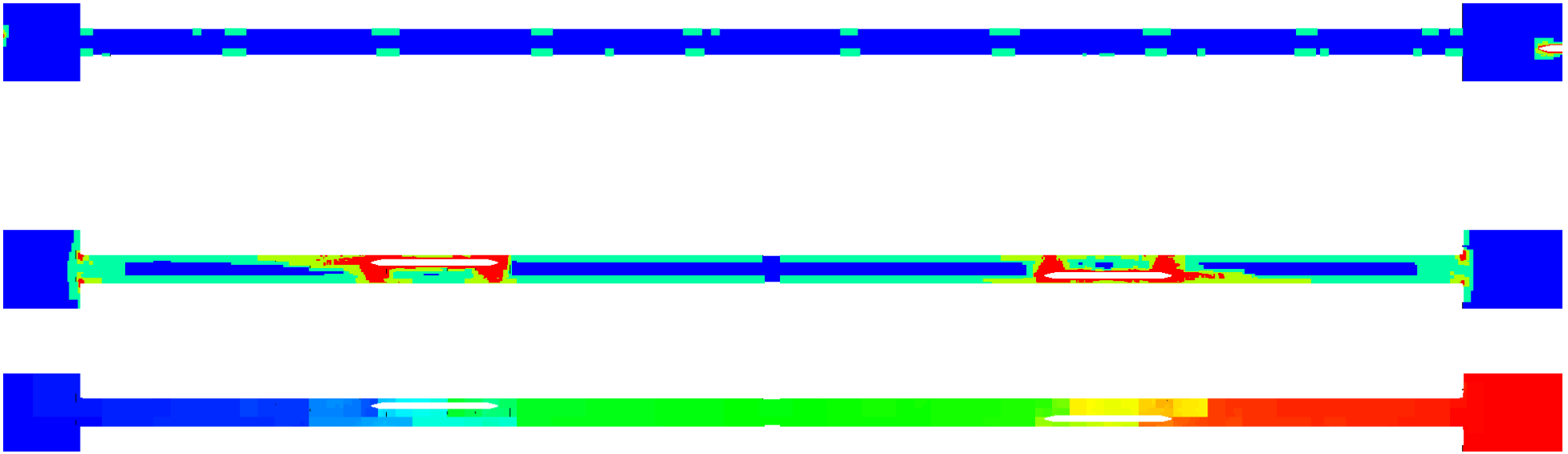


Tunnel shape



Setup with realistic tunnel shape

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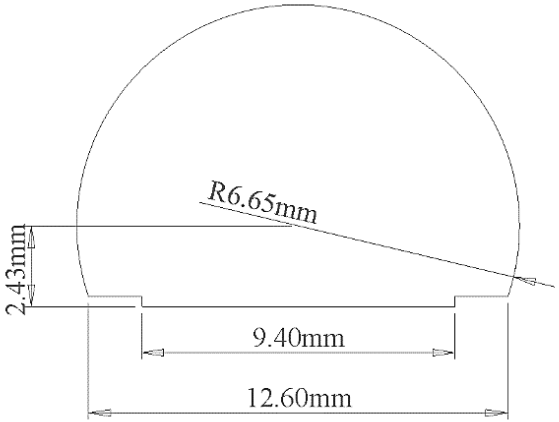
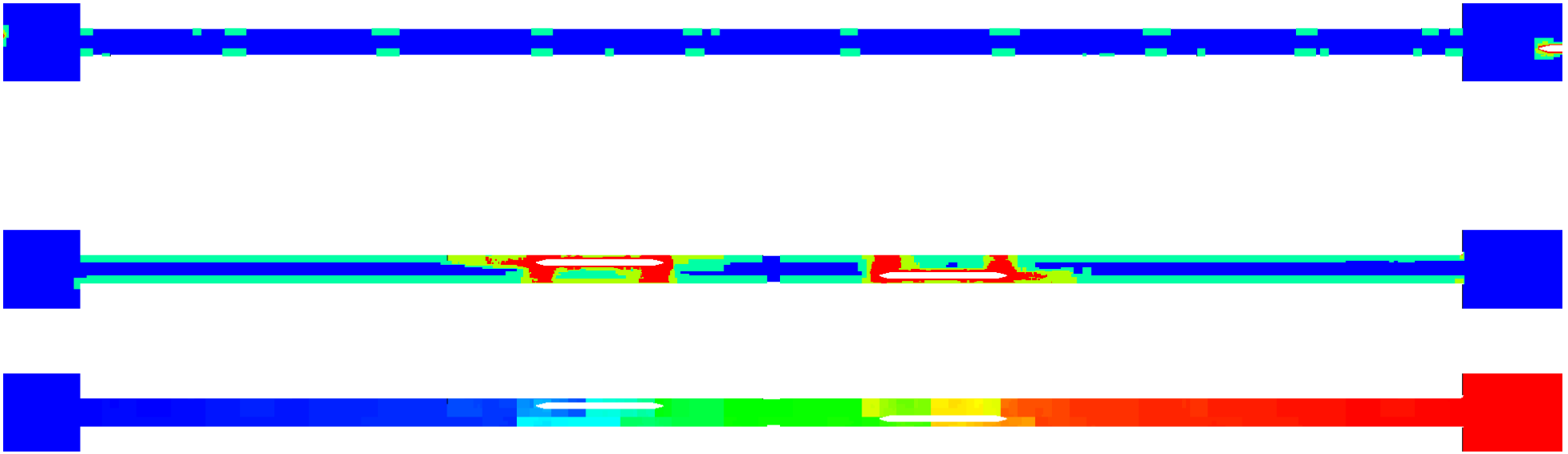
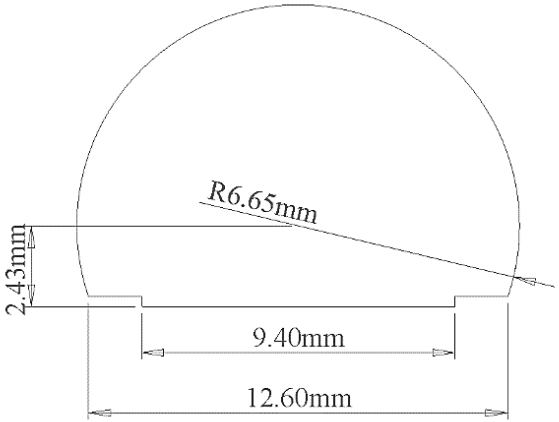


Tunnel shape

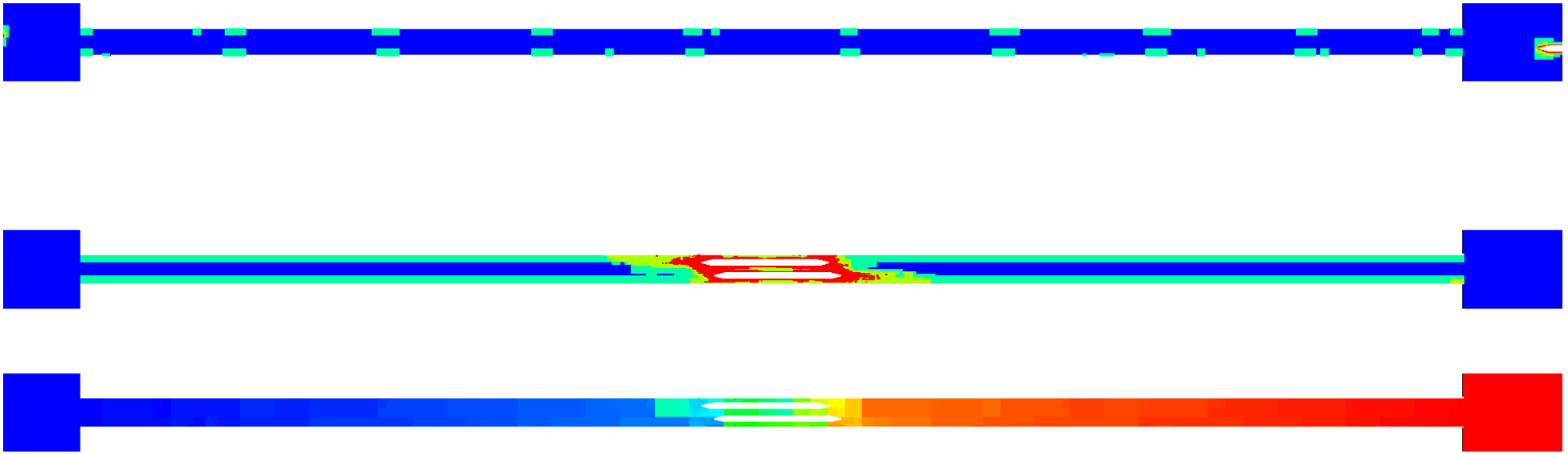


Setup with realistic tunnel shape

- Two NGT2 trains again at velocities $100\,\mathrm{m/s}$ and $-100\,\mathrm{m/s}$
- Prototype straight double track tunnel of $640\,\mathrm{m}$ length, initial distance between centers of trains $820\,\mathrm{m}$
- Base mesh of $1060 \times 36 \times 24$ for domain of $1060\,\mathrm{m} \times 36\,\mathrm{m} \times 24\,\mathrm{m}$, three levels refined by $r_{1,2,3} = 2$
- On 96 cores Intel Xeon E5-2670 $2.6\,\mathrm{GHz}$ a final $t_e = 5\,\mathrm{sec}$ was reached after $84,651\,\mathrm{sec}$ wall time, i.e., $2257\,\mathrm{h}$ CPU



Tunnel shape



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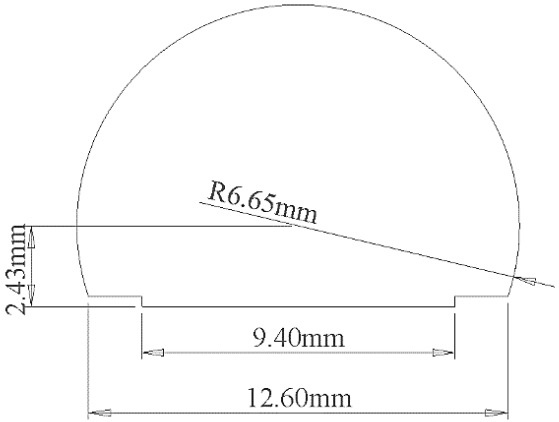

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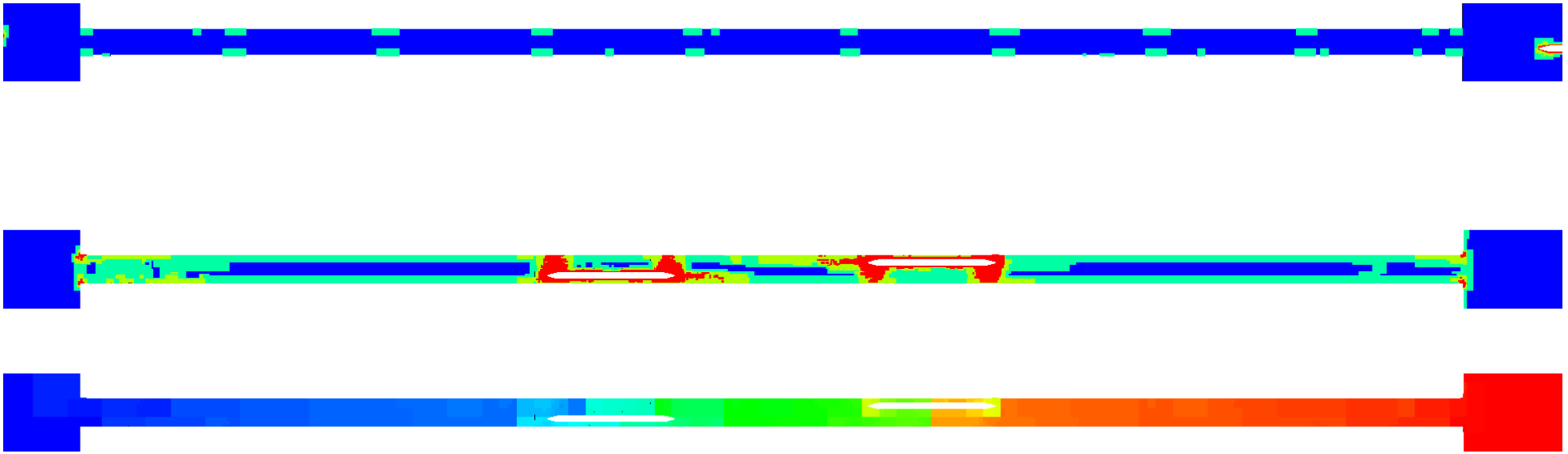
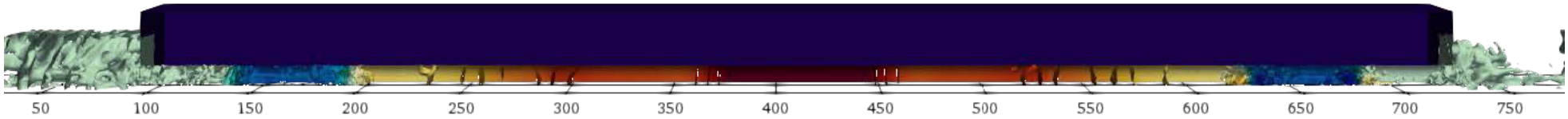
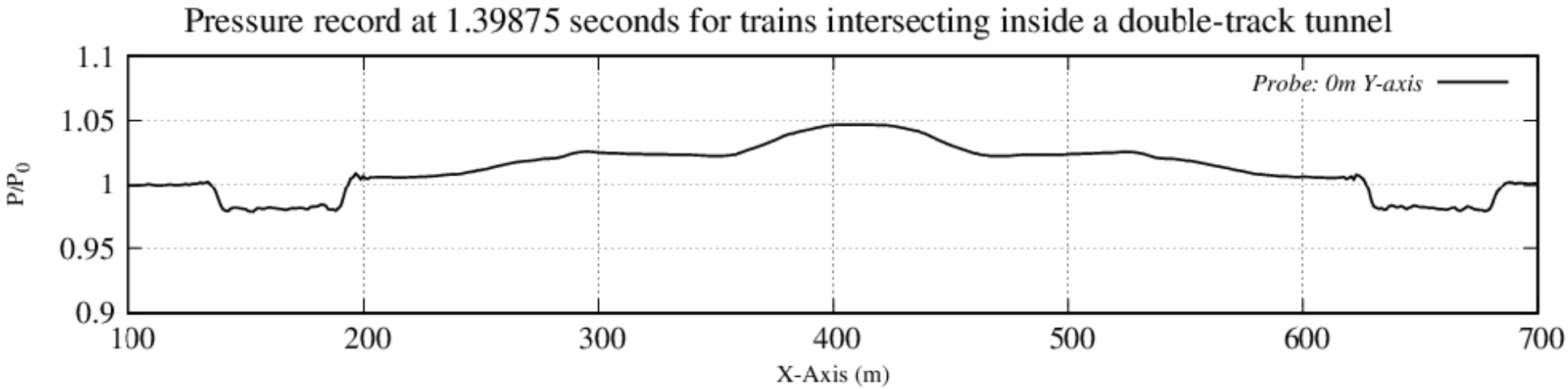
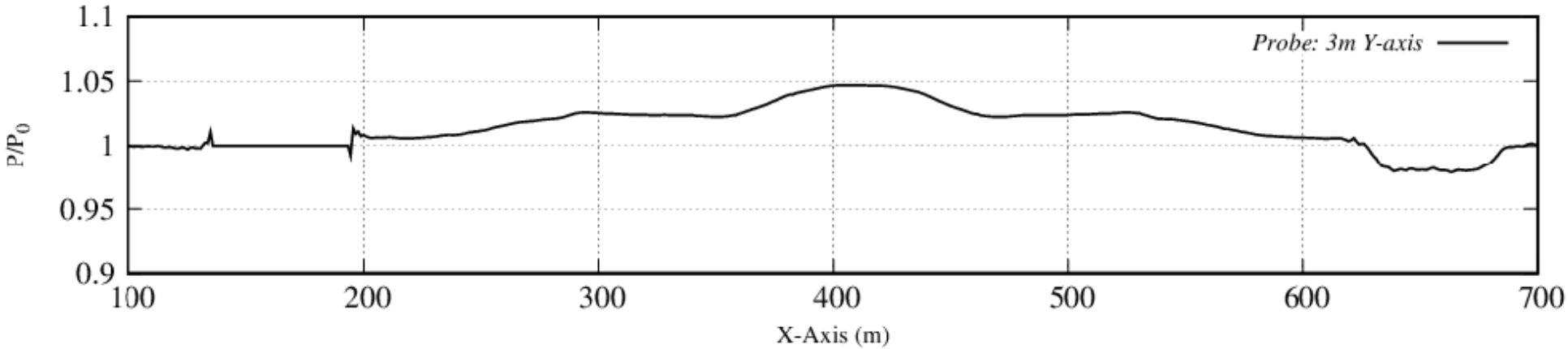


Pressure transects







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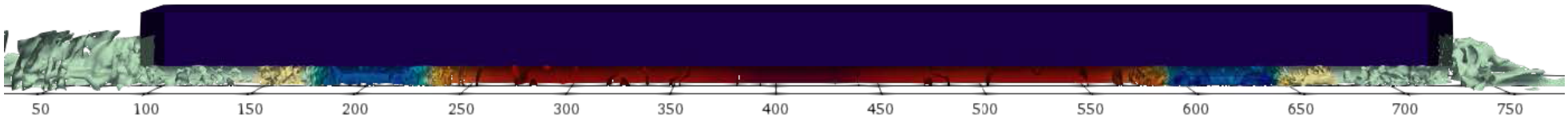

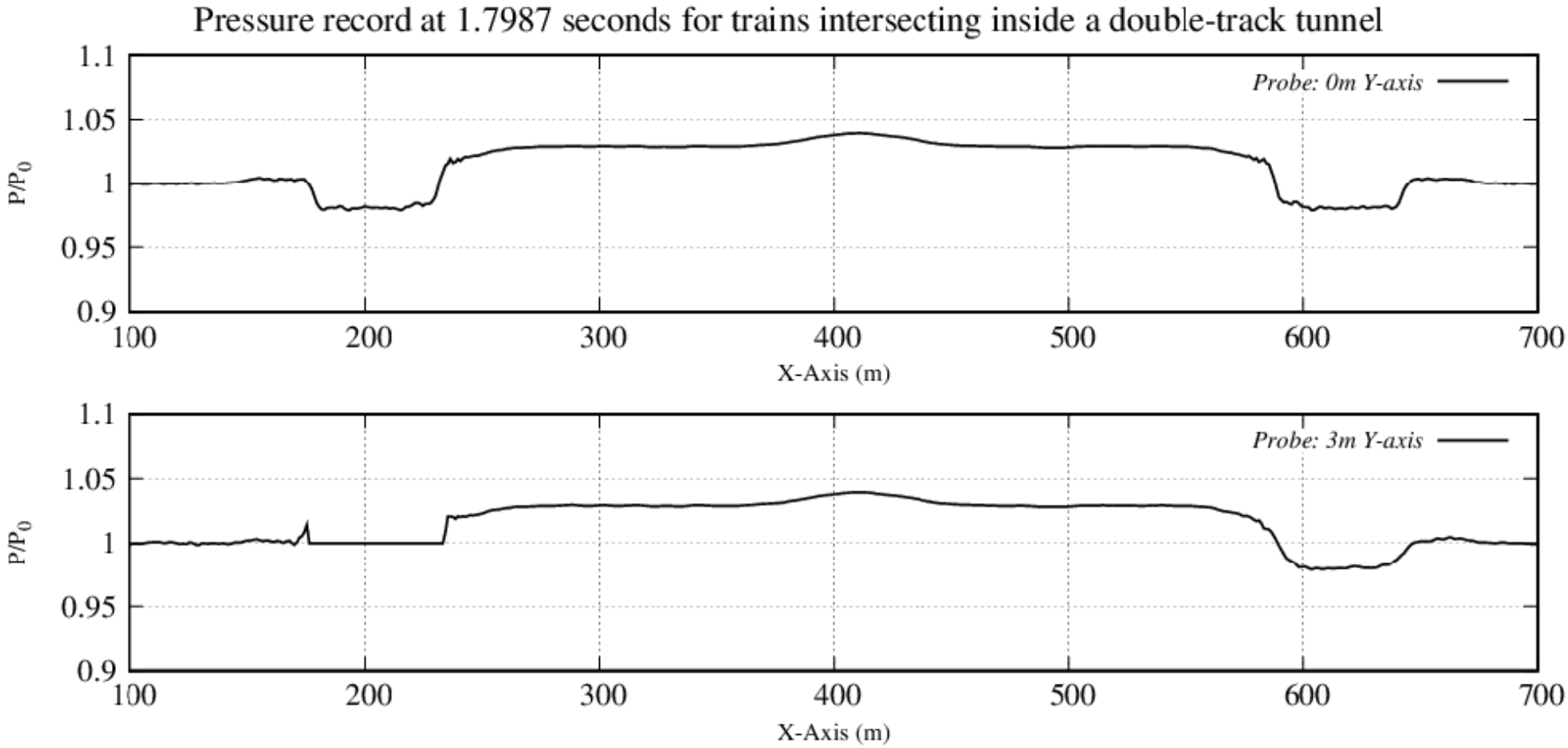

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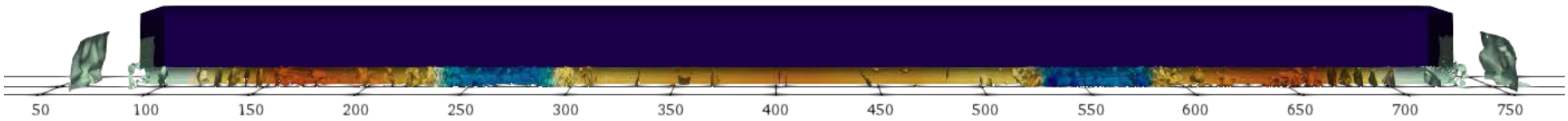
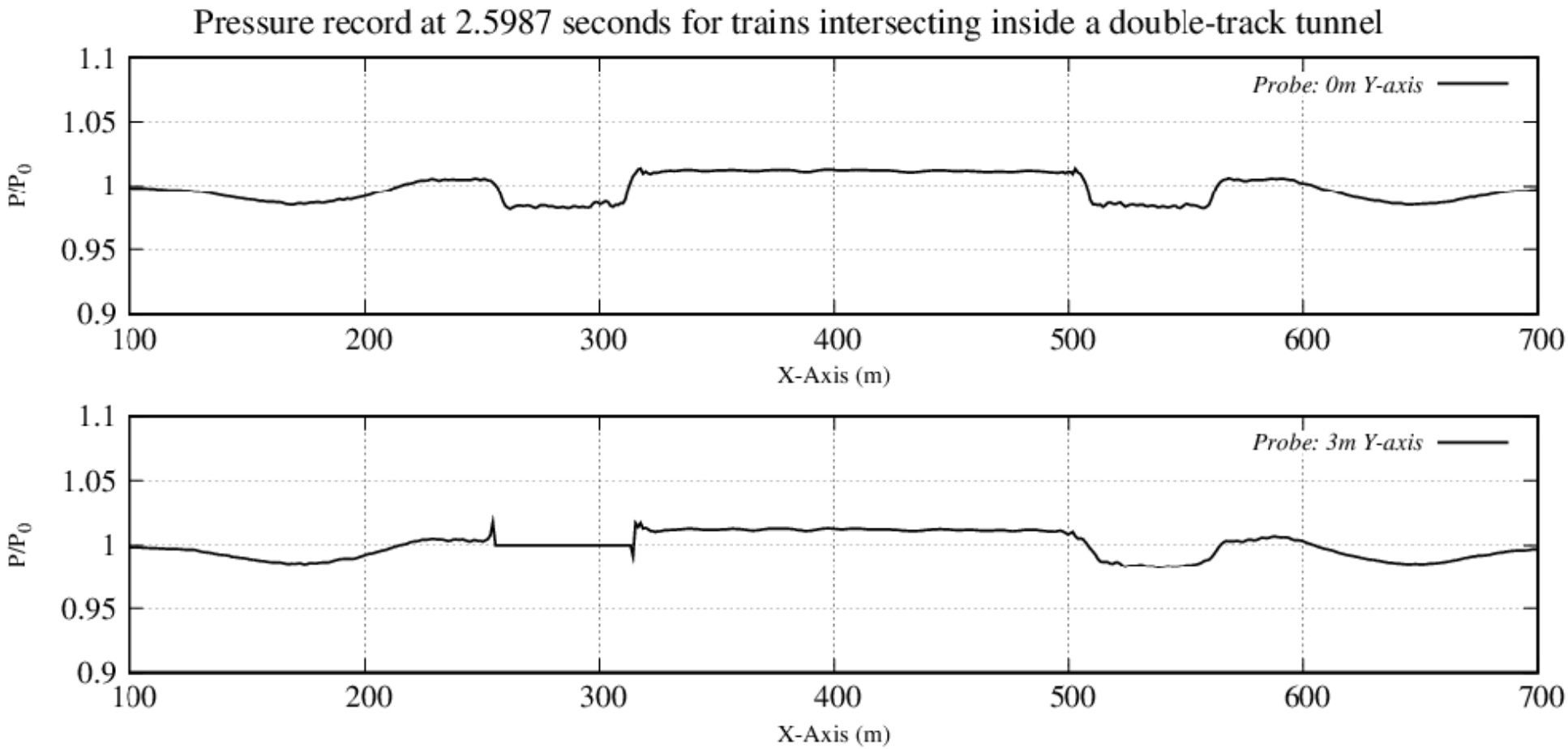
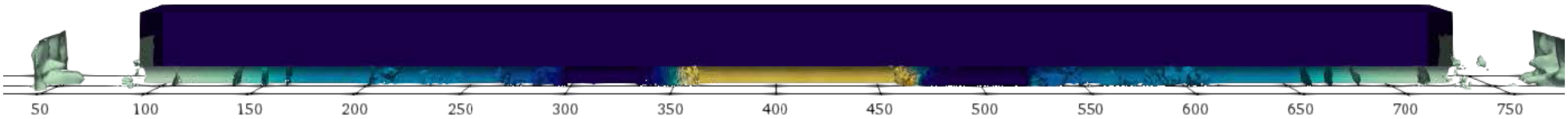
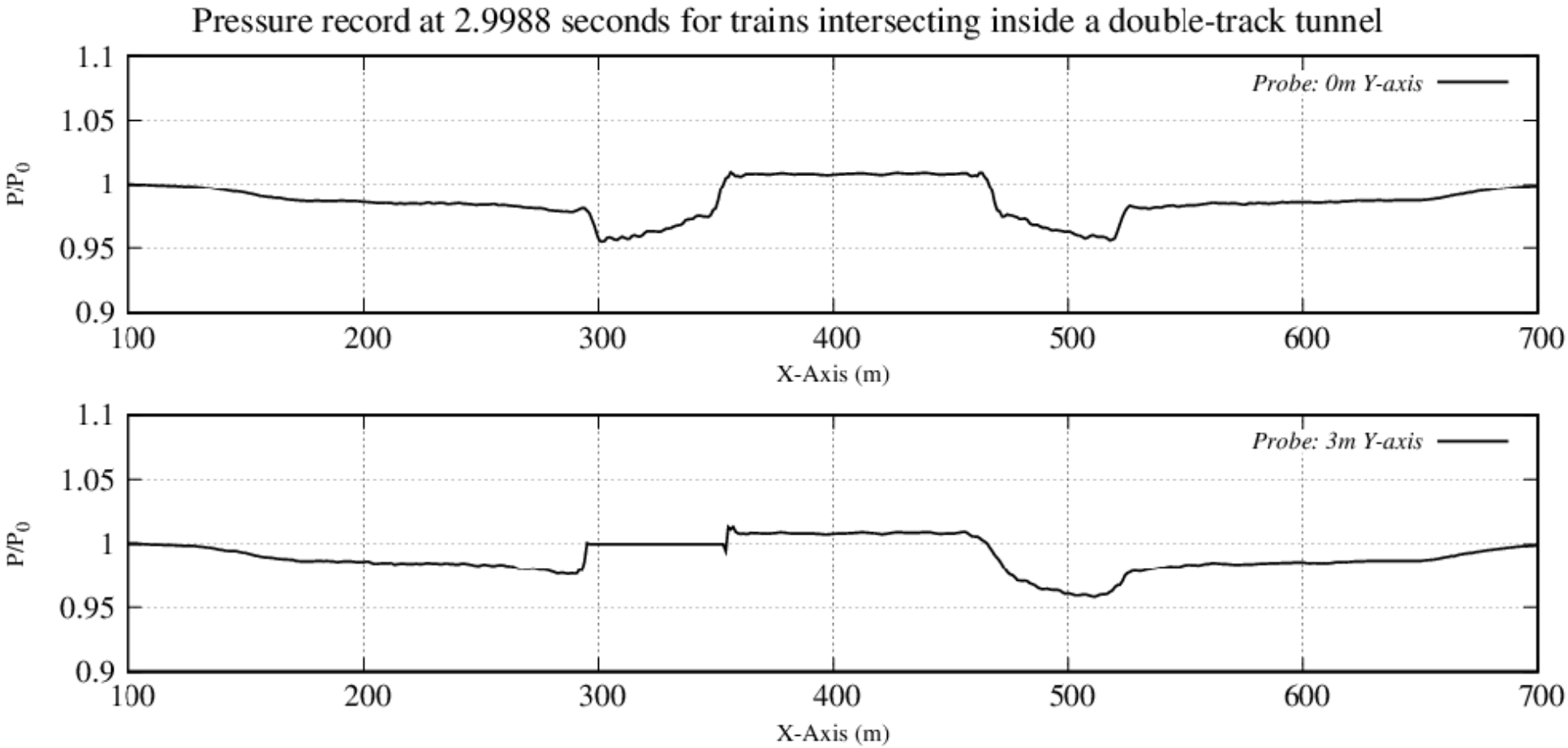


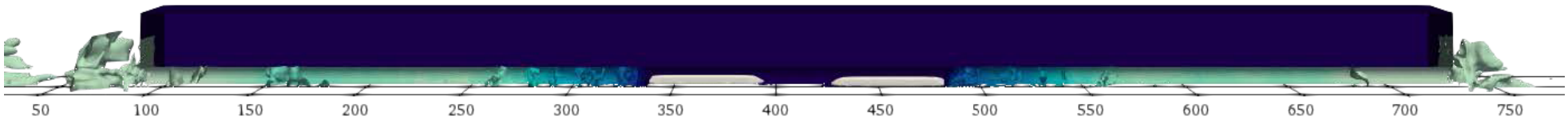


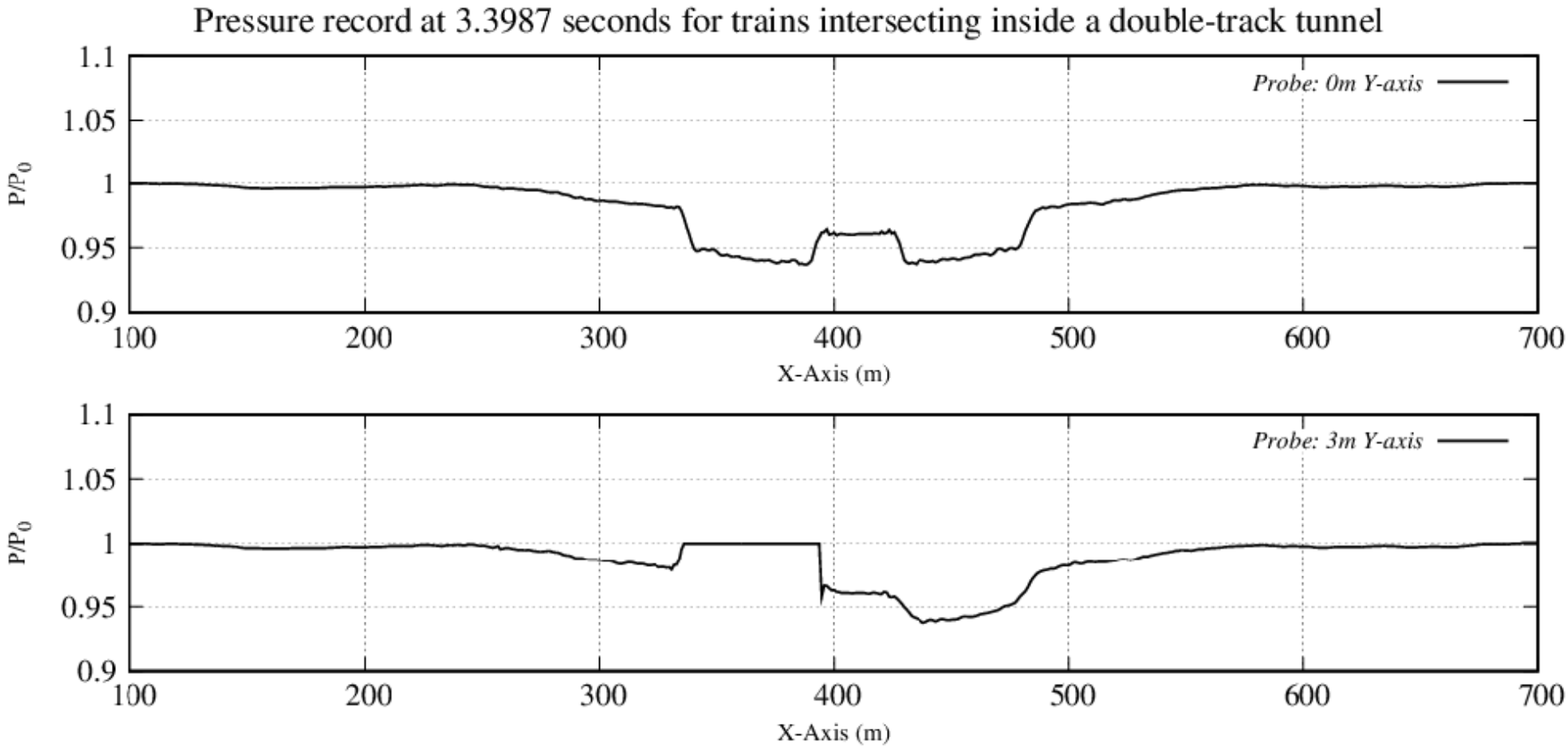
Pressure transects





Pressure transects





Conclusions

- A Cartesian embedded boundary method for compressible flows with block-based adaptive mesh refinement is a highly efficient and scalable prediction tool for pressure and shock waves created in front of high-speed trains. This gives predictions of maximal loading.

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- For predicting the flow around entire trains, the boundary layer growing over the train body needs to be considered.
- AMROC solvers for the compressible Navier-Stokes equations and even LES are already available, however, for this particular application a turbulent wall function on the embedded boundary first needs to implemented. Such a wall function is currently work-in-progress for the LBM-LES solver.

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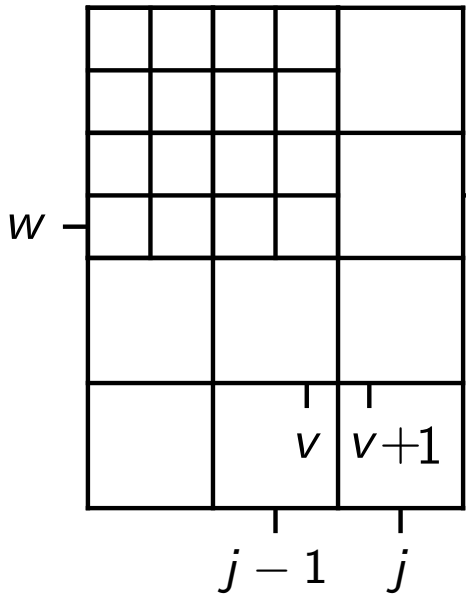
Conservative flux correction

Example: Cell j, k

$$\check{\mathbf{Q}}^{l}_{jk}(t+\Delta t_l) = \mathbf{Q}^{l}_{jk}(t) - \frac{\Delta t_l}{\Delta x_{1,l}} \left(\mathbf{F}^{l}_{j+\frac{1}{2},k} - \frac{1}{r^2_{l+1}} \sum_{\kappa=0}^{r_{l+1}-1} \sum_{\iota=0}^{r_{l+1}-1} \mathbf{F}^{l+1}_{v+\frac{1}{2},w+\iota}(t+\kappa\Delta t_{l+1}) \right) - \frac{\Delta t_l}{\Delta x_{2,l}} \left(\mathbf{G}^{l}_{j,k+\frac{1}{2}} - \mathbf{G}^{l}_{j,k-\frac{1}{2}} \right)$$

Correction pass:



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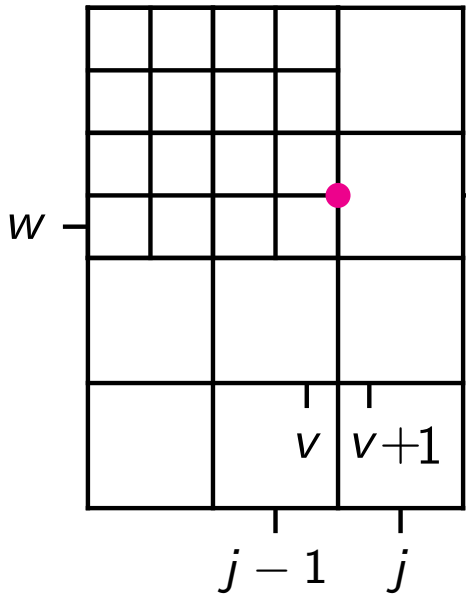
Correction pass:

1. $\delta\mathbf{F}^{l+1}_{j-\frac{1}{2},k} := -\mathbf{F}^{l}_{j-\frac{1}{2},k}$



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Example: Cell j, k

$$\check{\mathbf{Q}}^{l}_{jk}(t+\Delta t_l) = \mathbf{Q}^{l}_{jk}(t) - \frac{\Delta t_l}{\Delta x_{1,l}} \left(\mathbf{F}^{l}_{j+\frac{1}{2},k} - \frac{1}{r^2_{l+1}} \sum_{\kappa=0}^{r_{l+1}-1} \sum_{\iota=0}^{r_{l+1}-1} \mathbf{F}^{l+1}_{v+\frac{1}{2},w+\iota}(t+\kappa\Delta t_{l+1}) \right) - \frac{\Delta t_l}{\Delta x_{2,l}} \left(\mathbf{G}^{l}_{j,k+\frac{1}{2}} - \mathbf{G}^{l}_{j,k-\frac{1}{2}} \right)$$

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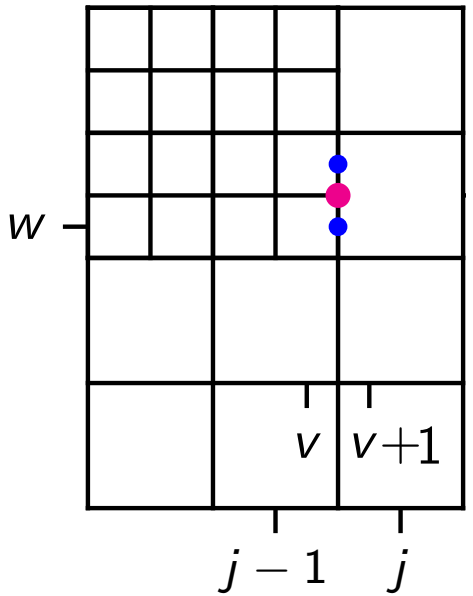

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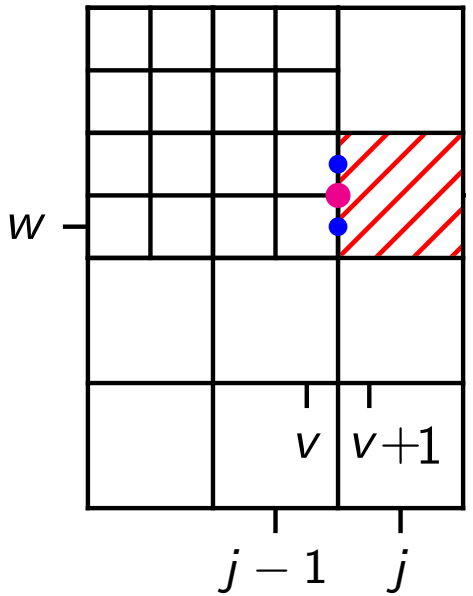
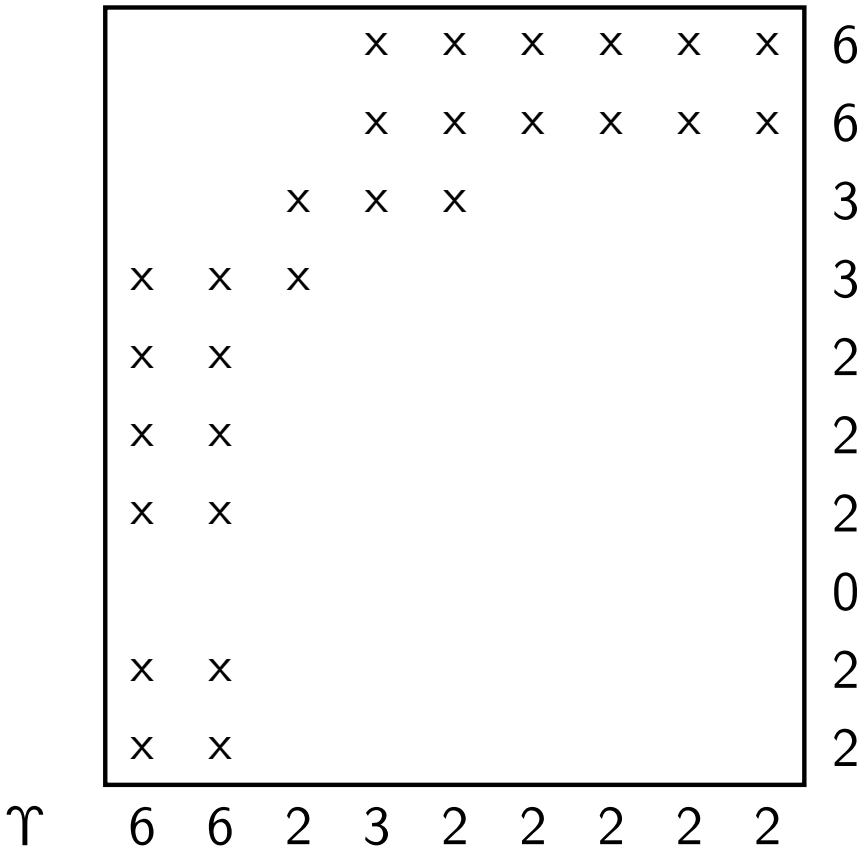
3. $\check{\mathbf{Q}}^l_{jk}(t+\Delta t_l) := \mathbf{Q}^l_{jk}(t+\Delta t_l) + \frac{\Delta t_l}{\Delta x_{1,l}} \delta\mathbf{F}^{l+1}_{j-\frac{1}{2},k}$



Clustering by signatures

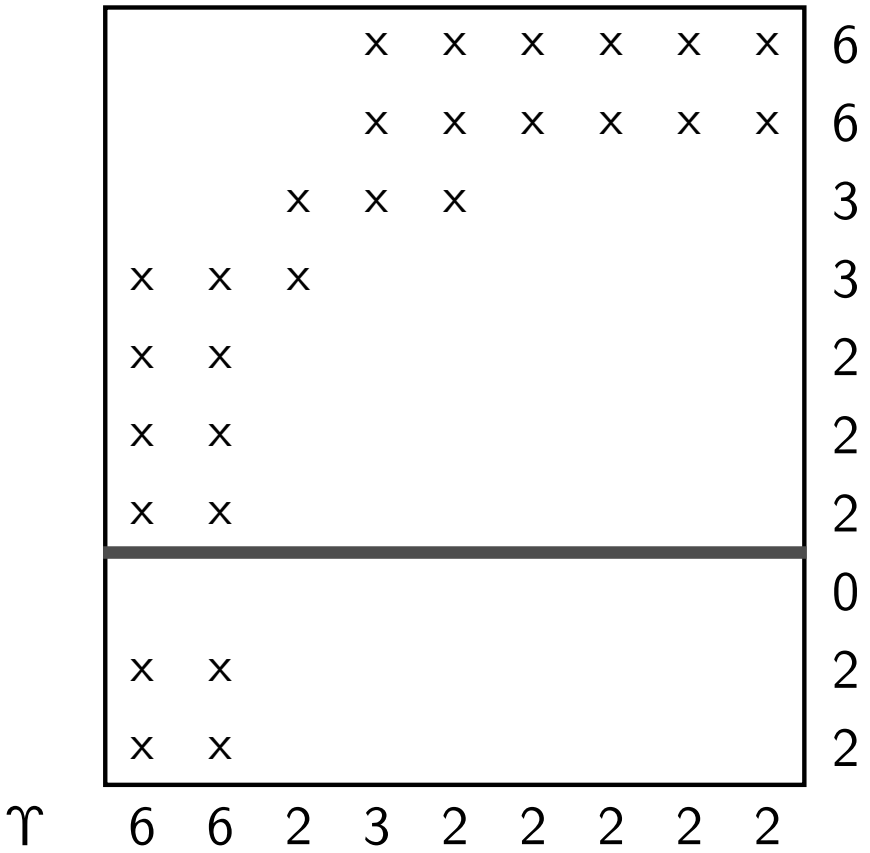


Υ Flagged cells per row/column

Δ Second derivative of Υ, $\Delta = \Upsilon_{\nu+1} - 2\,\Upsilon_{\nu} + \Upsilon_{\nu-1}$

Technique from image detection: [Bell et al., 1994], see also [Berger and Rigoutsos, 1991], [Berger, 1986]

Clustering by signatures



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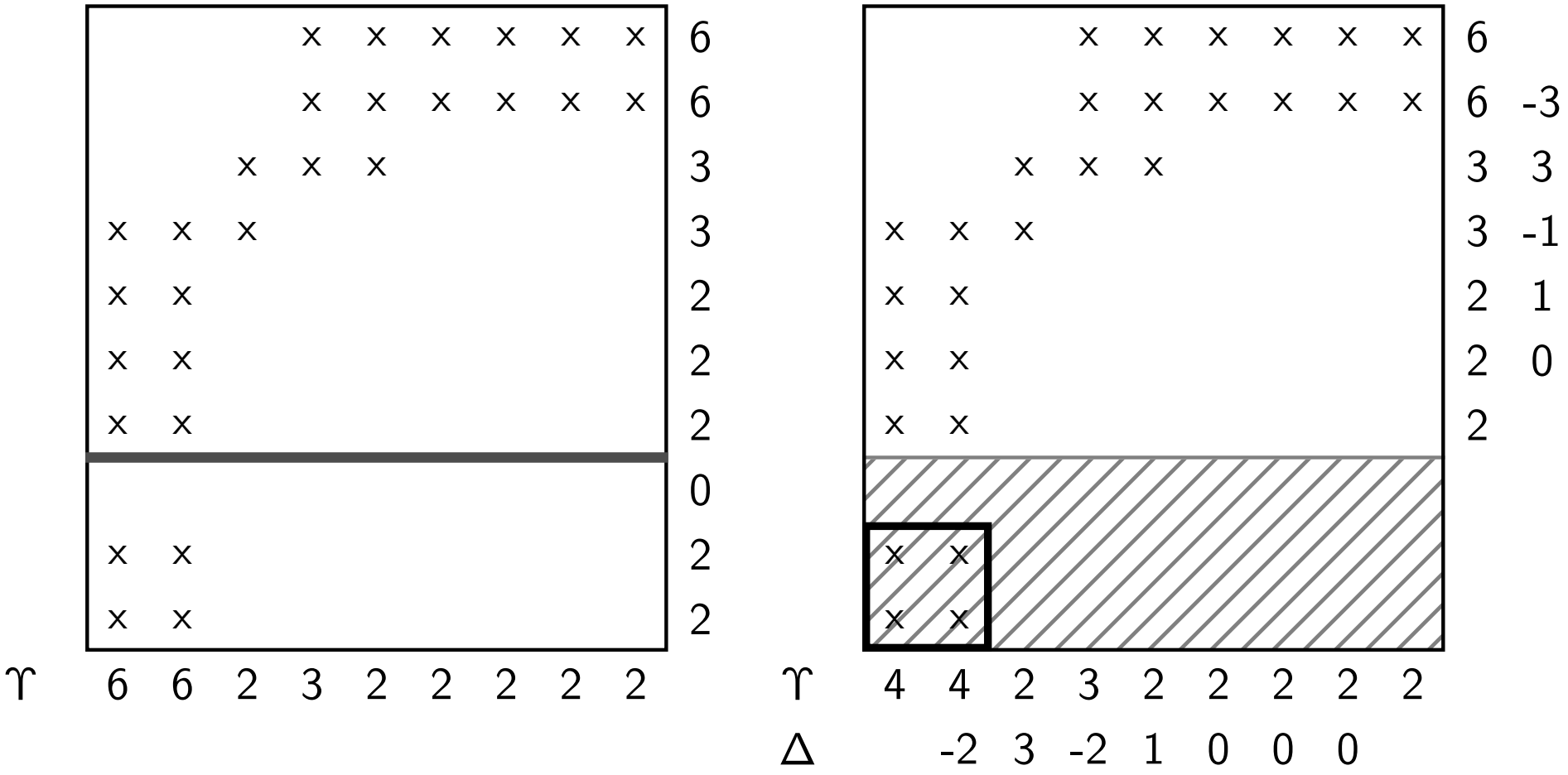

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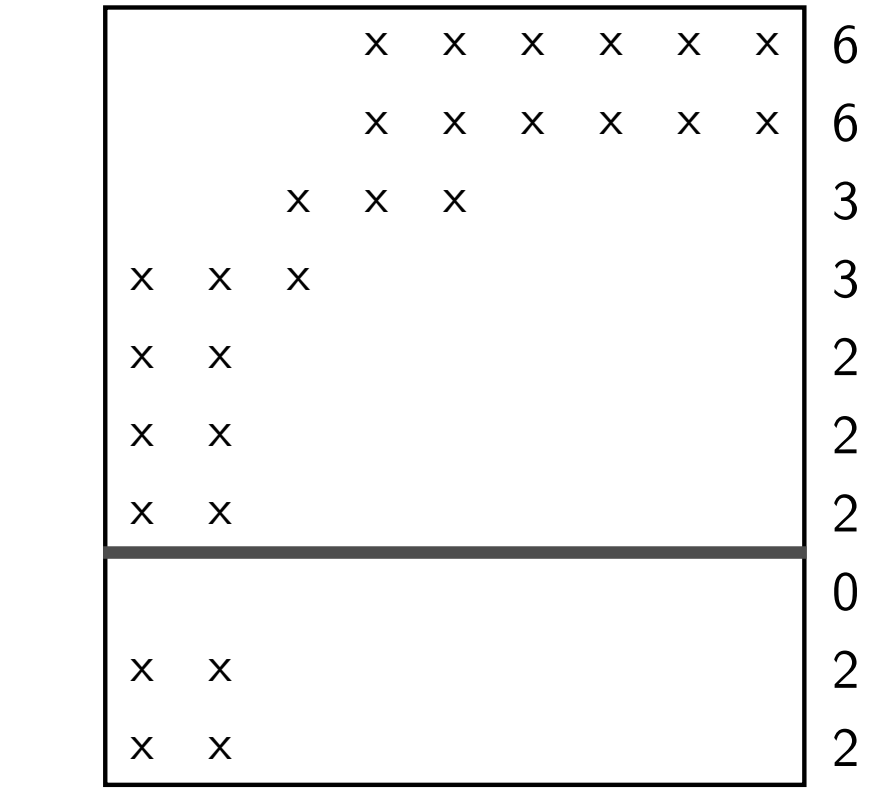

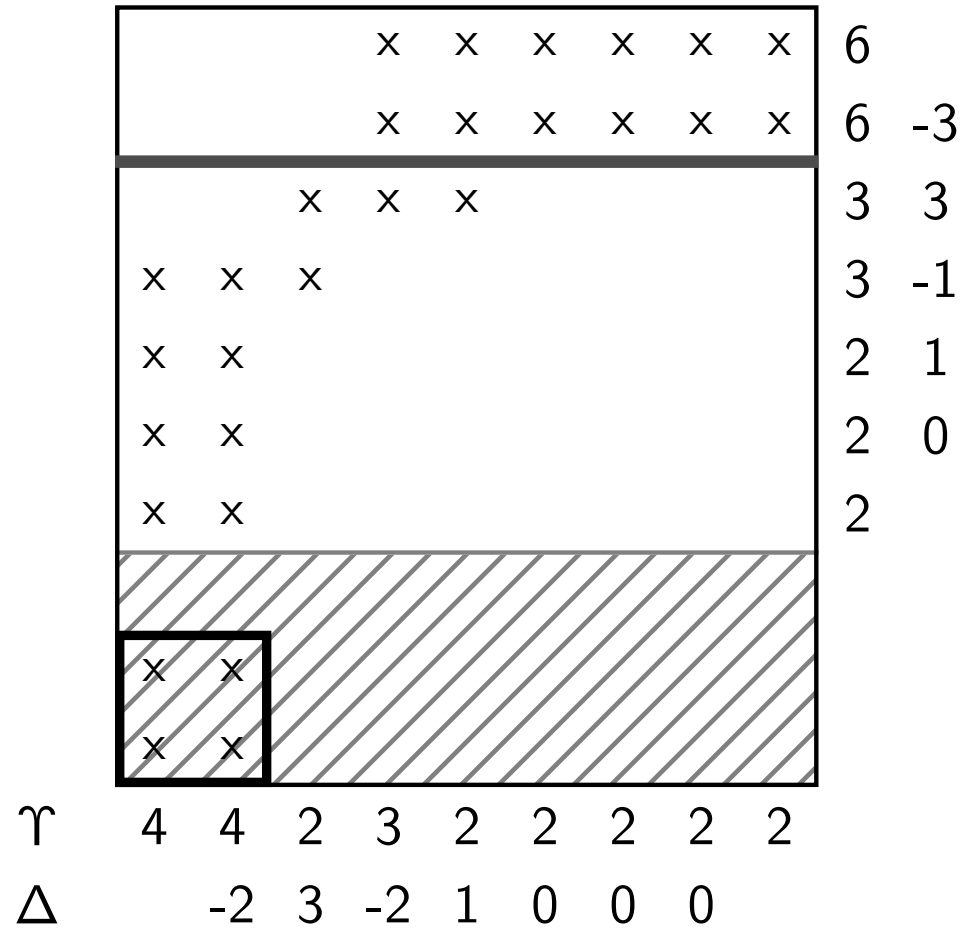

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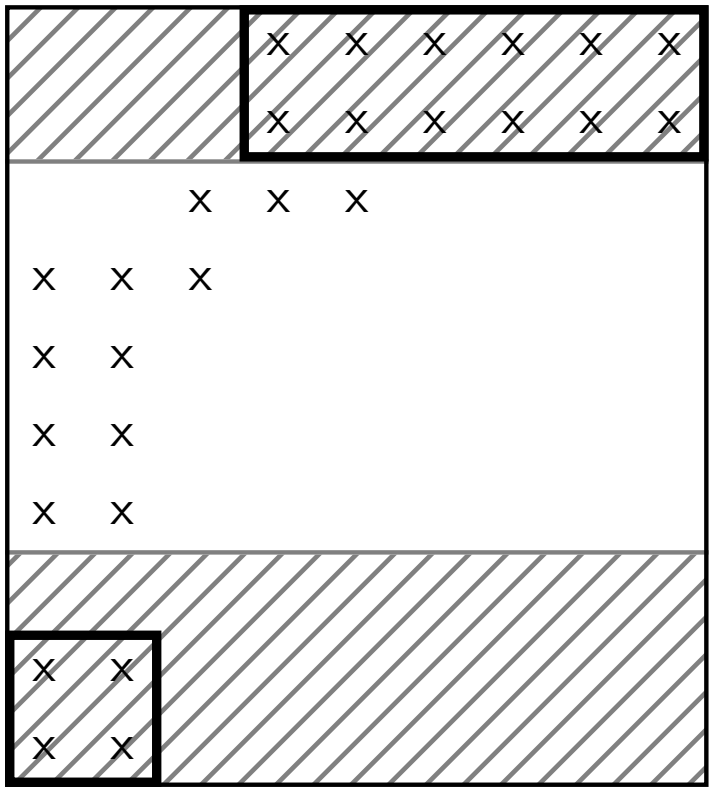
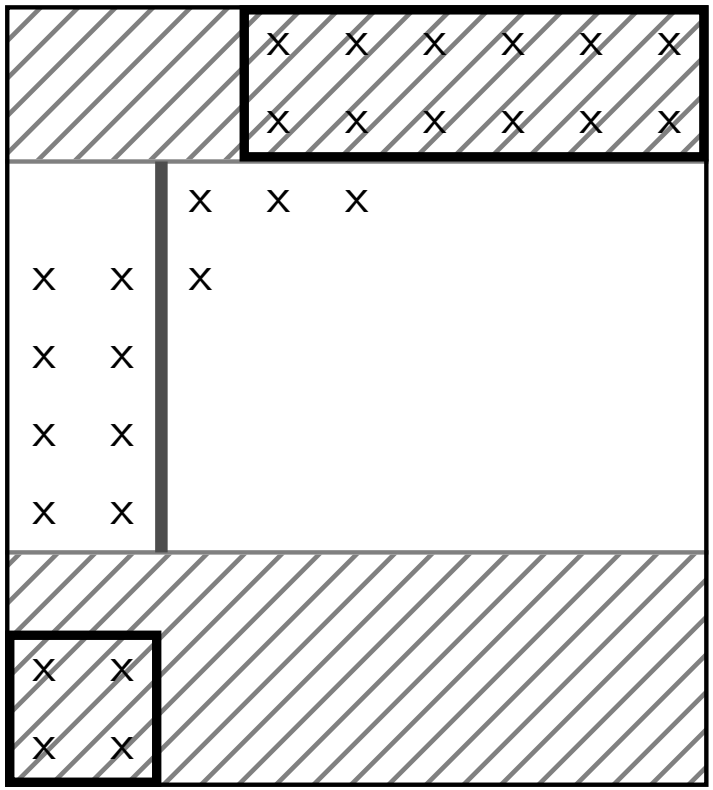


Υ	4	4	2	1	1		
Δ		-2	1				

Recursive generation of $\breve{G}_{l,m}$

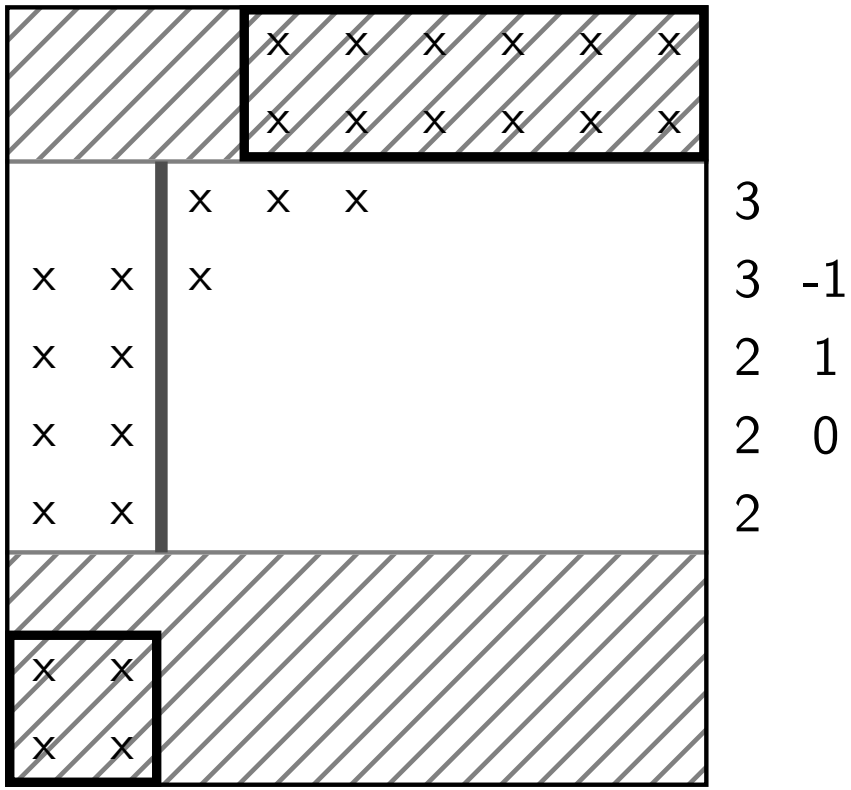
1. 0 in Υ
2. Largest difference in Δ
3. Stop if ratio between flagged and unflagged cell $> \eta_{tol}$

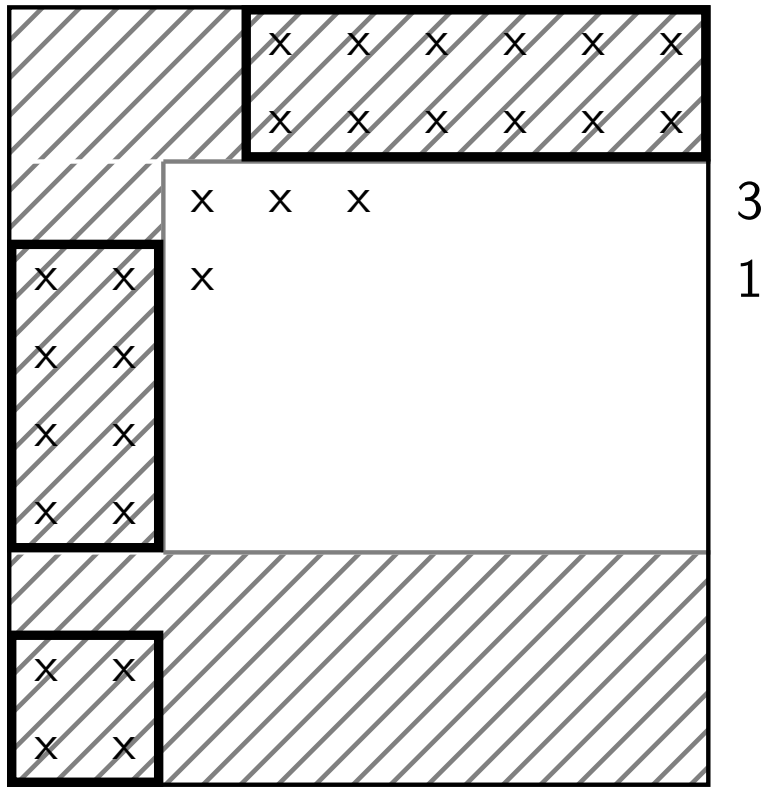


Υ	4	4	2	1	1		
Δ		-2	1				

Recursive generation of $\breve{G}_{l,m}$

1. 0 in Υ
2. Largest difference in Δ
3. Stop if ratio between flagged and unflagged cell $> \eta_{tol}$





Recursive generation of $\breve{G}_{l,m}$

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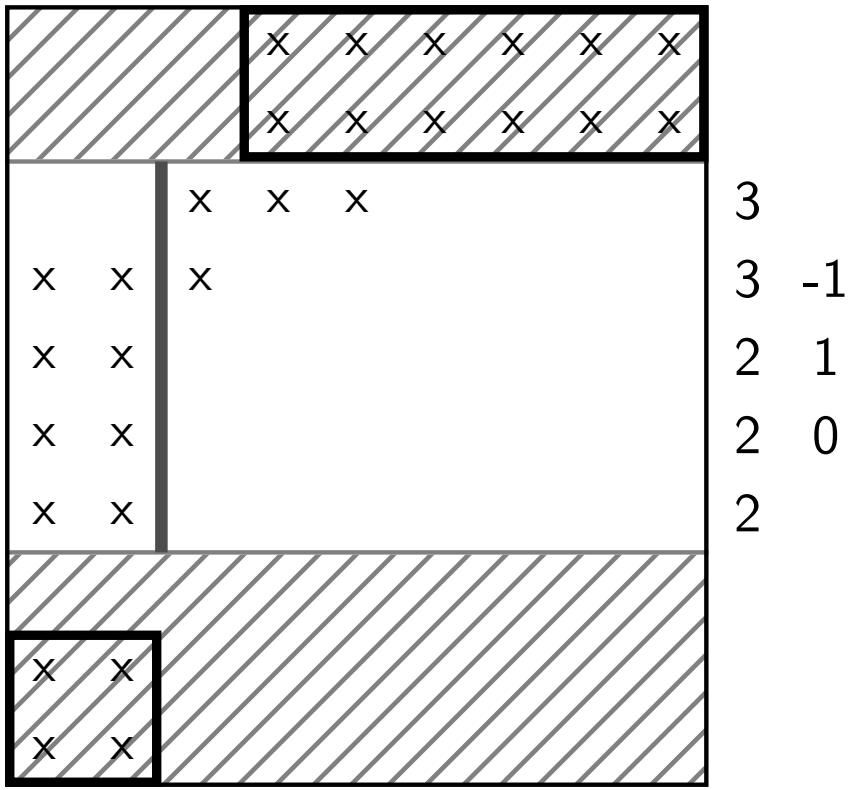

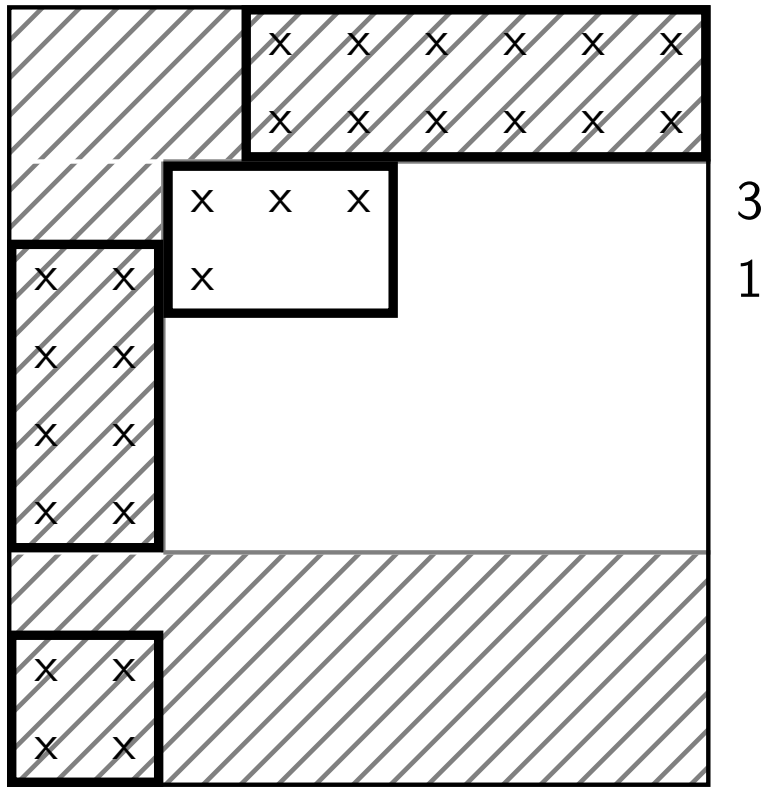

Recursive generation of $\breve{G}_{l,m}$

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Closest point transform algorithm

The signed distance φ to a surface $\mathcal{I}$ satisfies the eikonal equation [Sethian, 1999]

$$|\nabla\varphi| = 1 \quad \text{with} \quad \varphi\big|_{\mathcal{I}} = 0$$

Solution smooth but non-diferentiable across characteristics.

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- Geometric solution approach with plosest-point-transform algorithm [Mauch, 2003]

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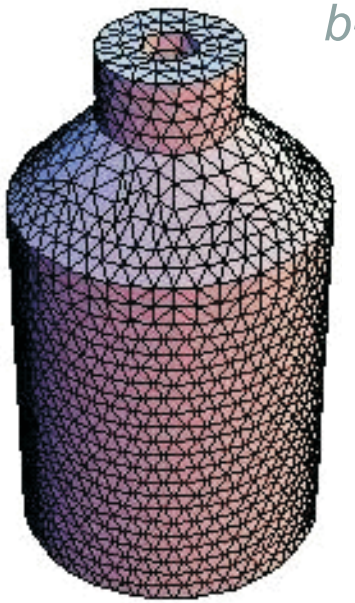
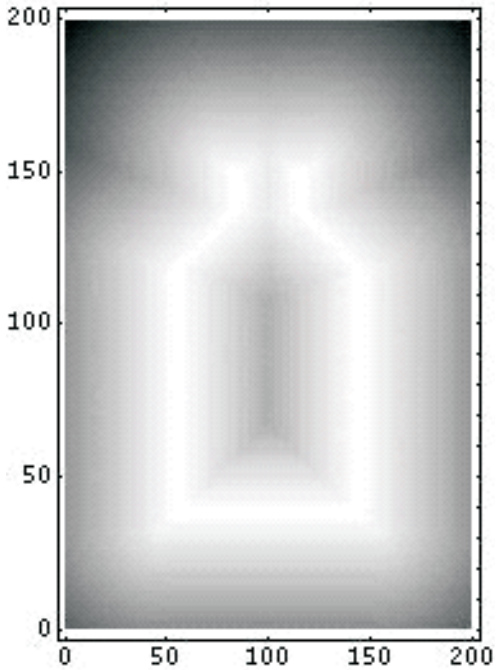
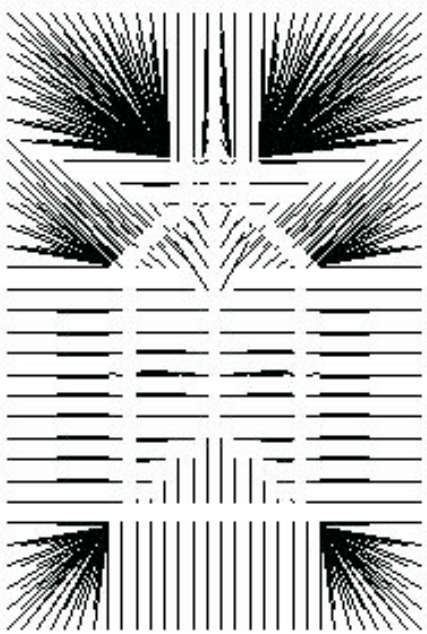


Surface mesh $\mathcal{I}$



Distance φ

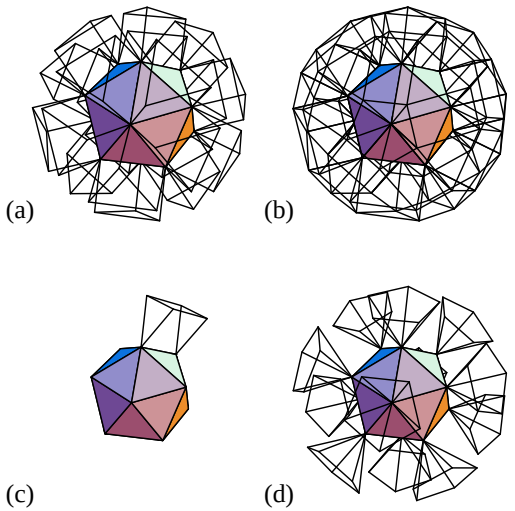


Normal to closest point

The characteristic / scan conversion algorithm

1. Build the characteristic polyhedrons for the surface mesh

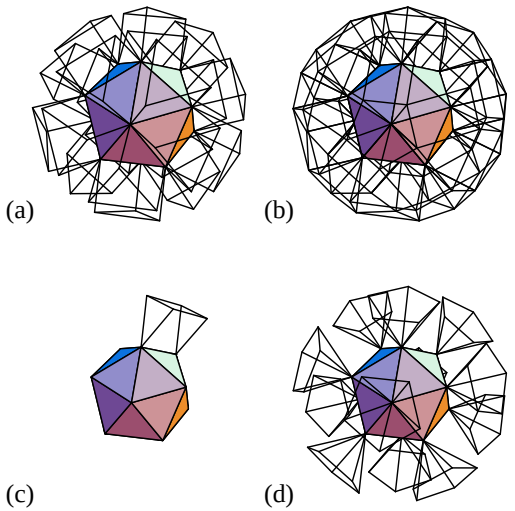
Characteristic polyhedra for faces, edges, and vertices



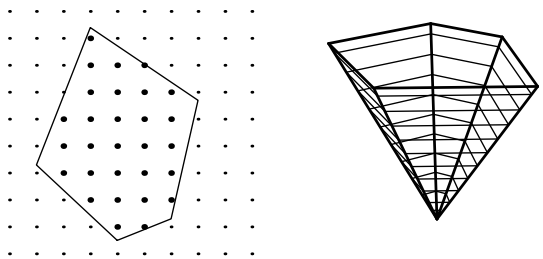
The characteristic / scan conversion algorithm

1. Build the characteristic polyhedrons for the surface mesh
2. For each face/edge/vertex
   2.1 Scan convert the polyhedron.

Characteristic polyhedra for faces, edges, and vertices



Slicing and scan conversion of apolygon



The characteristic / scan conversion algorithm

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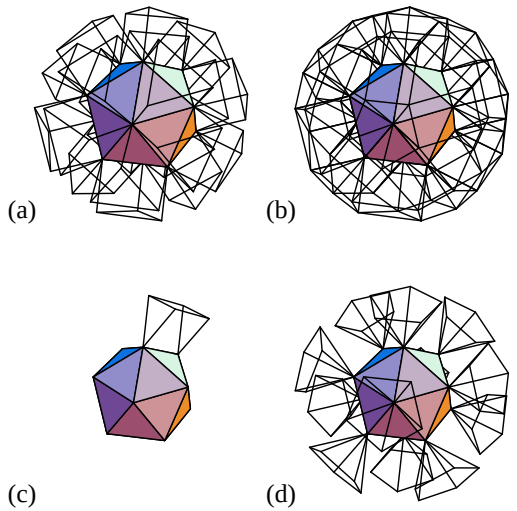
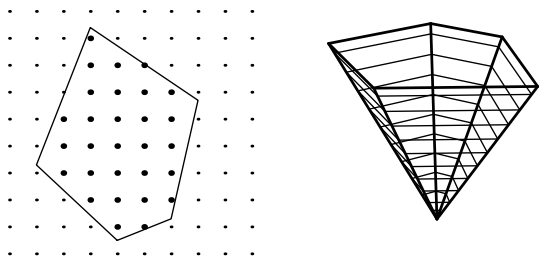
Characteristic polyhedra for faces, edges, and vertices



Slicing and scan conversion of apolygon



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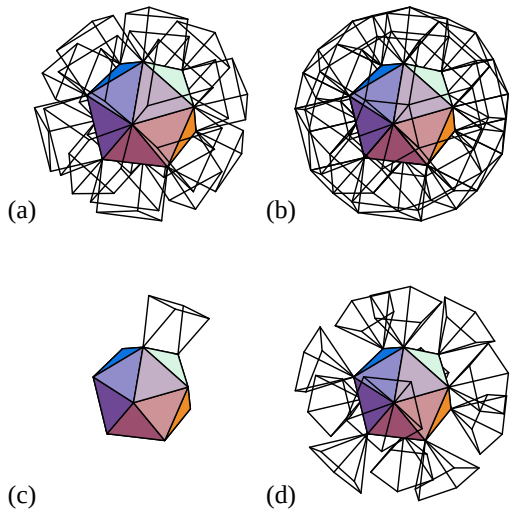
3. Computational complexity.
   - $O(m)$ to build the b-rep and the polyhedra.
   - $O(n)$ to scan convert the polyhedra and compute the distance, etc.

Characteristic polyhedra for faces, edges, and vertices



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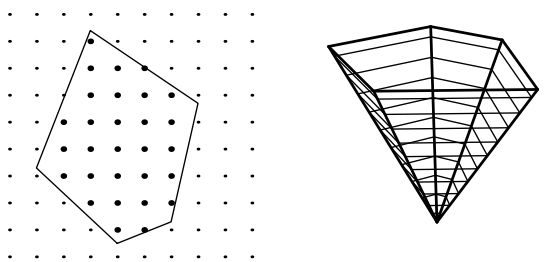

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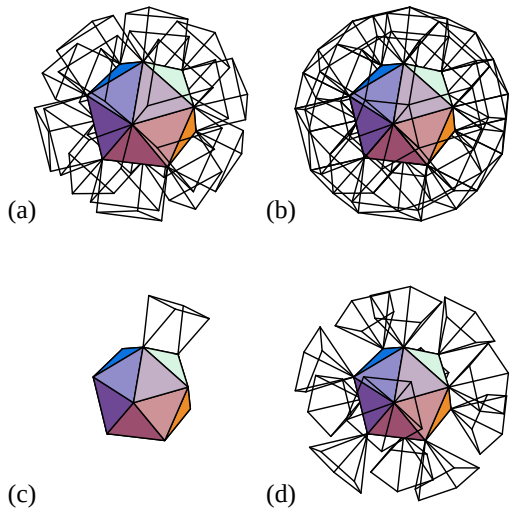
4. Problem reduction by evaluation only within specified max. distance

[Mauch, 2003], see also
[Deiterding et al., 2006]

Characteristic polyhedra for faces, edges, and vertices



Slicing and scan conversion of apolygon